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Experimental physics 3 for physicists

FAU Erlangen–Nuremberg, winter term 2018/19
https://mpl.mpg.de/en/leuchs/lehre/skripte.html

Problem sheet 2

08./09.11.2018

Problem 1: Frequency spectrum

10 vibrations of a sound signal with carrier frequency $f = 1\,$kHz are described by the function

$$u(t) = \begin{cases} u_0 \sin(2\pi f t) & 0 \le t \le 10\,\text{ms} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

a. Calculate the frequency spectrum of the signal;

b. Which is the minimum frequency range to be transmitted for a receiver to be able to estimate the original duration of the rectangular pulse?
Hint: Assume that the duration can be estimated if the spectrum is transmitted up to the first zero above the carrier frequency.

Problem 2: Spherical wave

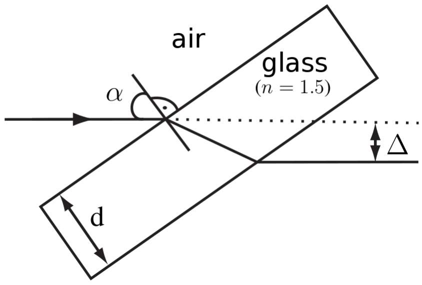
a. The scalar Helmholtz equation $(\triangle + k^2)\,u(r) = 0$ can be derived from the scalar wave equation in vacuum $\left(\frac{1}{\epsilon_0\mu_0}\triangle - \partial_t^2\right)u(r,t) = 0$ by considering a monochromatic ansatz $u(r,t) = u(r)\,\mathrm{e}^{i\omega t}$ and the dispersion relation $\omega^2 = c^2k^2$. A spherical wave is described by the expression $u(r) = \frac{\exp(ikr)}{r}$, for $r \neq 0$. Check that it is one of the solutions of the Helmholtz equation $(\triangle + k^2)\,u(r) = 0$, i.e. that it fulfills the equation under substitution.

   Hint: Due to the fact that the amplitude of a spherical wave depends only on absolute distance from the origin (in other words, possesses radial symmetry) it is convenient to perform the calculation in spherical coordinates. In this system the Laplace operator “operates” in the following way:
   $\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 f}{\partial\phi^2}$

b. Compare the wave emitted by a Hertzian dipole to a spherical wave. What is the difference? Why?

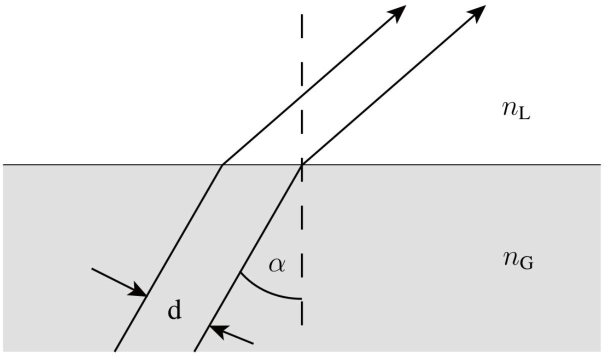
Problem 3: Plane-Parallel Plate

A beam of light passes through a plane-parallel plate of glass ($n = 1.5$) and undergoes a displacement Δ perpendicular to the propagation direction (see the Figure). Calculate this displacement as a function of the plate thickness d and the incident angle α.



Problem 4: Energy conservation at an interface

A light beam of the diameter d (see the Figure) falls at the incident angle $\alpha = 30°$ onto a plane interface between glass and air ($n_\mathsf{G} = 1.5$, $n_\mathsf{A} = 1$). The polarization is perpendicular to the figure plane.



a. Calculate the strength of the electric field in the reflection/transmission as a function of the incident field E_e.
b. How is the energy conserved in this case? Calculate the energy flux of the incident, reflected and transmitted beams. *Hint:* The energy flux (power) per unit area is given by the time-averaged modulus of the Poynting vector $\langle S \rangle = \frac{1}{2}\epsilon_0 c n E_0^2$.
c. What happens when the incident angle is increased to the value $\alpha = 45°$?