Optical computing by injection-locked Vertical-Cavity Surface-Emitting Laser (VCSEL)

Advanced Laser course presentation

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How does a computer work?

Algorithmic Logic Unit (ALU)[6]

An ALU is part of the Central Processing Unit (CPU) [. . .] and is capable of performing simple arithmetic operations [. . .] such as AND, OR, and the sum of two machine words by chance of logic gates.

Logic gate[6]

Logic gates are analog component, such as transistors. [. . .] Each gate has one or more digital inputs (signal representing 0 or 1) and computes as output some simple function of these inputs, such as AND or OR.

OR

x	y	output
0	0	0
1	0	1
0	1	1
1	1	1

AND

x	y	output
0	0	0
1	0	0
0	1	0
1	1	1

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Definition injection-locked VCSEL laser

injection-locked: enforcing operation of a laser on a certain optical frequency by injecting light with that frequency into the laser resonator[2]

vertical-cavity surface-emitting: a monolithic kind of semiconductor lasers with beam emission perpendicular to the wafer surface[4]

Normalizations

An injection locked laser can perform a normalization operation $y = p\frac{x}{\|x\|}$ that can be used to construct an programmable AND/OR logic gate, x and y represent complex amplitude of the master and slave.

Model for polarization dynamics in VCSEL (1)

Polarization light state emitted by VCSEL depends on: [3]

- angular momentum of the quantum states involved in the material transitions for emission and absorption
    - emission of a quantum of light with right (left) circular polarization corresponds to a transition in which the projection of the total material angular momentum on the direction of propagation changes by $+1$ (-1) in units of $\hbar$
- laser cavity
    - anisotropies, geometry and waveguiding effects

Model for polarization dynamics in VCSEL's (2)

- Angular momentum depends on the electron-hole recombination process in a semiconductor
  - electron states of the conduction band have a total angular momentum quantum number $J_z = \pm\frac{1}{2}$
  - electron states of the heavy hole band have a total angular momentum quantum number $J_z = \pm\frac{3}{2}$ (in bulk matter)
- If neglecting transitions from the conduction to the light hole band, transitions allowed in surface emitting quamtum-well are those in which $\Delta J_z = \pm 1$

$\Rightarrow$ only two transitions allowed

- $J_z = -\frac{1}{2} \rightarrow J_z = -\frac{3}{2}$ (right circular polarized light)
- $J_z = +\frac{1}{2} \rightarrow J_z = +\frac{3}{2}$ (left circular polarized light)

Model for polarization dynamics in VCSEL's (3)

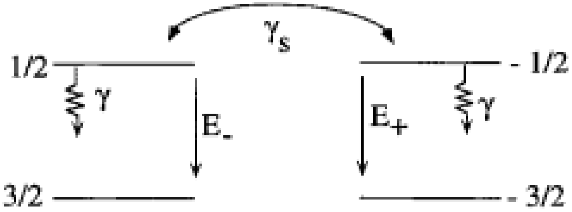

Figure: Four-level model for polarization dynamics in VCSEL [3]

Maxwell-Bloch equations for VCSEL (1)

$$\dot{E}_x = -\gamma_c E_x - i\gamma_c \alpha E_x + \gamma_c (1 + i\alpha)(NE_x + inE_y) \quad (1)$$

$$\dot{E}_y = -\gamma_c E_y - i\gamma_c \alpha E_y + \gamma_c (1 + i\alpha)(NE_y - inE_x) \quad (2)$$

$$\dot{N} = -\gamma[N(1 + |E_x|^2 + |E_y|^2) - \mu + in(E_y Ex^* - E_x E_y^*)] \quad (3)$$

$$\dot{n} = -\gamma_s n - \gamma[n(|E_x|^2 + |E_y|^2) + iN(E_y E_x^* - E_x E_y^*)] \quad (4)$$

where: [5]

α = linewidth enhancement factor

γ_c = decay rate of E in the cavity

γ = decay rate of the total carrier number

γ_s = decay rate related to the electron angular momentum

μ = pumping rate of the laser

Maxwell-Bloch equations for VCSEL (2)

(3) and (4):

$$\dot{N} = -\gamma[\underbrace{N(1 + |E_x|^2 + |E_y|^2)}_{\text{absorption}} - \underbrace{\mu}_{\text{pumping}} + \underbrace{in(E_y Ex^* - E_x E_y^*)}_{\text{stimulated emission}}]$$

$$\dot{n} = -\underbrace{\gamma_s n}_{\text{loss}} - \gamma[\underbrace{n(|E_x|^2 + |E_y|^2)}_{\text{loss}} + \underbrace{iN(E_y E_x^* - E_x E_y^*)}_{\text{spontaneous emission}}]$$

where: [5]

γ_c = decay rate of the electric field in the cavity

γ = decay rate of the total carrier number

γ_s = decay rate related to the electron angular momentum

μ = pumping rate of the laser

Maxwell-Bloch equations for VCSEL (3)

- Assumptions:
  - master electric fields with varying amplitude $\bar{u}$ which is a sum of input electric fields with the slowly varying amplitudes $\bar{e}_k$: $\bar{u} = \sum_k \bar{e}_j$
  - $\bar{e}_k$ and $\bar{E}$ have the same carrier frequencies
  - weak coupling: $\|\bar{u}\| \ll \|\bar{E}\|$

$\Longrightarrow$ (1) and (2):

$$\dot{E}_x = -\gamma_c E_x - i\gamma_c \alpha E_x + \gamma_c (1 + i\alpha)(NE_x + inE_y) + u_x \qquad (1)$$

$$\dot{E}_y = -\gamma_c E_y - i\gamma_c \alpha E_y + \gamma_c (1 + i\alpha)(NE_y - inE_x) + u_y \qquad (2)$$

$$0 = -\gamma_c E_x - i\gamma_c \alpha E_x + \gamma_c (1 + i\alpha)(N E_x + i n E_y) + u_x \quad (1)$$
$$0 = -\gamma_c E_y - i\gamma_c \alpha E_y + \gamma_c (1 + i\alpha)(N E_y - i n E_x) + u_y \quad (2)$$
$$0 = -\gamma [N(1 + |E_x|^2 + |E_y|^2) - \mu + i n (E_y E x^* - E_x E_y^*)] \quad (3)$$
$$0 = -\gamma_s n - \gamma [n(|E_x|^2 + |E_y|^2) + i N (E_y E_x^* - E_x E_y^*)] \quad (4)$$

$$(4) \Leftrightarrow n = i\gamma N \frac{E_y E_x^* - E_x E_y^*}{\gamma_s + \gamma \left\| E \right\|^2}$$

- If linearly polarized field: $E_y E_x^* - E_x E_y^* = 0 \implies n = 0$ (4)

Steady-state (2)

$$0 = -\gamma_c E_x - i\gamma_c \alpha E_x + \gamma_c(1+i\alpha)(NE_x + inE_y) + u_x \quad (1)$$

$$0 = -\gamma_c E_y - i\gamma_c \alpha E_y + \gamma_c(1+i\alpha)(NE_y - inE_x) + u_y \quad (2)$$

$$0 = N(1 + \|E\|^2) - \mu \quad (3)$$

$$(1) \wedge (2) \Leftrightarrow E_{x,y} = \frac{u_{x,y}}{\gamma_c + i\gamma_c\alpha - \gamma_c(1+i\alpha)N} = \frac{u_{x,y}}{\gamma_c(1+i\alpha)(1-N)} \quad (5)$$

$$\Leftrightarrow \|\bar{E}\|^2 = \frac{\|\bar{u}\|^2}{\gamma_c^2(1+\alpha^2)(1-N)^2} \quad (6)$$

$$(3) \Leftrightarrow N = \frac{\mu}{1+\|E\|^2} \quad (7)$$

$$(6) \wedge (7) \Leftrightarrow N\left(1 + \frac{\|\bar{u}\|^2}{\gamma_c^2(1+\alpha^2)(1-N)^2}\right) - \mu = 0 \quad (8)$$

Steady-state (3)

$$N\left(1+\frac{\|\bar{u}\|^2}{\gamma_c^2(1+\alpha^2)(1-N)^2}\right)-\mu=0 \tag{8}$$

- Let's set $N \equiv 1+\xi$:

$$\xi^3+\xi^2(1-\mu)+\xi\frac{\|\bar{u}\|^2}{\gamma_c^2(1+\alpha^2)}+\frac{\|\bar{u}\|^2}{\gamma_c^2(1+\alpha^2)}=0 \tag{8}$$

- Let's set $\Delta\xi' \equiv \dfrac{\|\bar{u}\|^2}{\gamma_c^2(1+\alpha^2)}$
- Approx. with a second degree Taylor polynomial in neighborhood of $\xi=0$:

$$\xi^2(1-\mu)+\xi\Delta\xi'+\Delta\xi'=0 \tag{8}$$

Steady-state (4)

$$(8)\ \xi^2(1-\mu)+\xi\Delta\xi'+\Delta\xi' = 0 \implies \begin{cases} \xi^{(1)} = \dfrac{-\Delta\xi' + \sqrt{\Delta\xi'^2 - 4(1-\mu)\Delta\xi'}}{2(1-\mu)} \\ \xi^{(2)} = \dfrac{-\Delta\xi' - \sqrt{\Delta\xi'^2 - 4(1-\mu)\Delta\xi'}}{2(1-\mu)} \end{cases}$$

- Let's set:

$$\begin{cases} \xi^{(1)} = \xi + \Delta\xi \\ \xi^{(2)} = -\xi + \Delta\xi \end{cases} \text{,where } \Delta\xi = -\frac{\Delta\xi'}{2(1-\mu)} \wedge \xi = \frac{\sqrt{\Delta\xi'^2 - 4(1-\mu)\Delta\xi'}}{2(1-\mu)} \quad (9)$$

- Yet $\Delta\xi' \equiv \dfrac{\|\bar{u}\|^2}{\gamma_c^2(1+\alpha^2)} \wedge \gamma_c \gg 0 \Rightarrow \gamma_c^{-4} \ll \gamma_c^{-2} \Rightarrow \Delta\xi'^2 \ll \Delta\xi'$,

$$(9)\ \Rightarrow \xi \approx \frac{\sqrt{-4(1-\mu)\Delta\xi'}}{2(1-\mu)} = -\frac{\sqrt{4(\mu-1)\Delta\xi'}}{2(\mu-1)} = -\frac{\sqrt{\Delta\xi'}}{\sqrt{\mu-1}}$$

Steady-state (5)

- Let's set:

$$\begin{cases} N^{(1)} = 1 + \xi^{(1)} \\ N^{(2)} = 1 + \xi^{(2)} \end{cases} \tag{10}$$

- Let's consider left and right hand polarized electric fields with:

$$\begin{cases} E_+ = \frac{1}{\sqrt{2}}(E_x + iE_y) \\ E_- = \frac{1}{\sqrt{2}}(E_x - iE_y) \end{cases} \wedge \begin{cases} u_+ = \frac{1}{\sqrt{2}}(u_x + iu_y) \\ u_- = \frac{1}{\sqrt{2}}(u_x - iu_y) \end{cases} \tag{11}$$

$$(1) \wedge (2) \wedge (11) \Rightarrow \dot{E}_{\pm} = -\gamma_c E_{\pm} - i\gamma_c \alpha E_{\pm} + \gamma_c (1 + i\alpha)(N \pm n) E_{\pm} + u_{\pm} \quad (12)$$

- Yet $n = 0$ for linearly polarized light,

$$(12) \Leftrightarrow \dot{E}_{\pm} = -\gamma_c (1 + i\alpha)(1 - N^{(k)}) E_{\pm} + u_{\pm} = -\beta^{(k)} E_{\pm} + u_{\pm}$$

where

$$\begin{cases} N^{(k)} = N, k \in \{1, 2\} \\ \beta^{(k)} = (1 + i\alpha)\gamma_c (1 - N^k) = -(1 + i\alpha)\gamma_c \xi^{(k)} \end{cases}$$

- Let's set:

$$(12) \Rightarrow E_{\pm}(t) = \frac{1}{\beta^{(k)}} u_{\pm} + e^{-\beta^{(k)} t} \left(E_{\pm}(0) - \frac{1}{\beta^{(k)}} u_{\pm} \right), t > 0 \quad (13)$$

Steady-state (7)

$$\begin{cases} \beta^{(1)} = \beta + \Delta\beta \\ \beta^{(2)} = -\beta + \Delta\beta \end{cases} \quad \text{where} \begin{cases} \beta = -(1+i\alpha)\gamma_c \xi \\ \Delta\beta = -(1+i\alpha)\gamma_c \Delta\xi \end{cases}$$

- Yet $\xi < 0 \wedge \Delta\xi \ll 1 \Rightarrow Re(\beta^{(1)}) > 0 \wedge Re(\beta^{(2)}) < 0$ $\implies$ only the solution corresponding to $k = 1$ is stable

$$\Rightarrow \xi^{(1)} \approx \xi = -\frac{\|\bar{u}\|}{\gamma_c \sqrt{1+\alpha^2}\sqrt{\mu - 1}}$$

$$\Rightarrow N^{(1)} \approx N = 1 - \frac{\|\bar{u}\|}{\gamma_c \sqrt{1+\alpha^2}\sqrt{\mu - 1}}$$

$$\Rightarrow (5) \Leftrightarrow E_{x,y} = \frac{u_{x,y}}{\gamma_c(1+i\alpha)} \frac{1}{\xi} = \frac{u_{x,y}}{\|\bar{u}\|} \frac{\sqrt{1+\alpha^2}\sqrt{\mu-1}}{(1+i\alpha)}$$

$$= \frac{u_{x,y}}{\|\bar{u}\|} e^{i\theta} \sqrt{\mu - 1} \text{ where } \theta = \arctan\alpha$$

MAOT master programme in advanced optical technologies

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Normalization

$$\bar{E} = \frac{\bar{u}}{\|\bar{u}\|} e^{i\theta} \sqrt{\mu - 1} \tag{14}$$

- amplified normalization of complex vector $\bar{u} \in \mathbb{C}^2$
- adopt the linearly polarized master electric field
- amplitude of the slave return to a constant

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Scheme

- AND / OR gate consists of an injection-locked laser with three optical inputs
- Inputs A & B & a bias X are combined at the laser and are assumed to carry equal optical power
- Signals are phase encoded such that the binary numbers of $\begin{cases} 0 \\ 1 \end{cases}$ correspond to a phase shift of $\begin{cases} 0 \\ \pi \end{cases}$ and relate to an electric field amplitude of $\begin{cases} -1 \\ +1 \end{cases}$, noted $E = \begin{pmatrix} E^v \\ E^h \end{pmatrix}$ where $E^v, E^h \in \mathbb{C}$

Simple constructions for OR and AND gates

- Let's consider steady state of one laser with incoming signals E_1, E_2 and a bias signal E_0 (b correspond to bits 0 and 1):

$$E_j = \begin{pmatrix} (2b_j - 1) \\ 0 \end{pmatrix} = \begin{pmatrix} a_j \\ 0 \end{pmatrix}$$

where $j \in \{0, 1, 2\} \wedge b_j \in \{0, 1\} \Rightarrow a_j \in \{-1, 1\}$

- Let's assume that signals are phase-controlled and it is possible to mix theme so that the light going in a laser has the amplitude:

$$E_{in} = E_1 + E_2 + E_0$$

$$\Rightarrow E_{out}(t) = \frac{E_{in}(t)}{|E_{in}(t)|} = \begin{pmatrix} a_{out} \\ 0 \end{pmatrix}$$

Example: AND gate, $b_0 = 0$

- $a_{out} = a_1 + a_2 + a_3$ and let's set $b_{out} = \frac{1}{2}(a_{out} + 1) \cdot H(a_{out})$

b_0	b_1	b_2	a_0	a_1	a_2	a_{out}	b_{out}	phase
0	0	0	-1	-1	-1	-3	0	π
0	0	1	-1	-1	+1	-1	0	π
0	1	0	-1	+1	-1	-1	0	π
0	1	1	-1	+1	+1	+1	1	0

Experience setup (1)

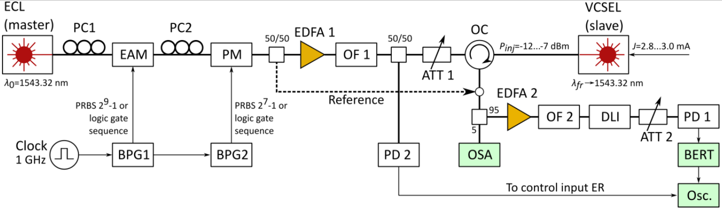

Figure: Schematic illustration of the measurement setup[5]

- The master *ECL-external-cavity-laser*, *EAM-electro-absorption-modulator* and *PM-phase-modulator* produce the desired encoding equivalent to the superposition of each bit input that result in multilevel phase-modulated signal with relative power levels of 1 and 9dB (*ER-extinction-ration* of 9.5dB) and the phase shifts of 0 and π.

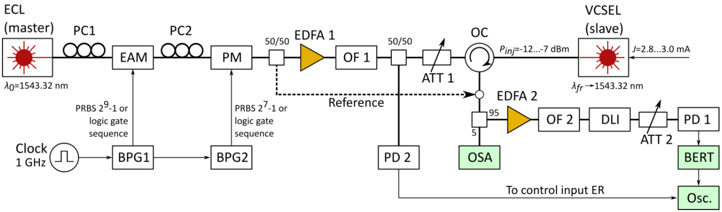

Figure: Schematic illustration of the measurement setup[5]

- *EDFA 1-erbium-doped-fibre-amplifier*, *OF 1-optical-filter* and *ATT 1-variable attenuator* control and optimize the optical power of the seeding signal.
- signal coupled to the slave *VCSEL* via an *OC-optical-circulator* which directed the returning signal into *EDFA 2* and *OF 2*.

Experience setup (2)

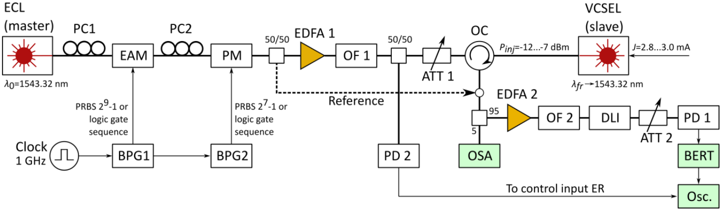

Figure: Schematic illustration of the measurement setup[5]

- *DLI-delay-line-interferometer*, *BERT-bit-error-rate-tester* and *Osc.-oscilloscope* measure the *BER-bit-error-ration* and the phase shift.

- BER of 10^{-6}
- No change in the wavelength or signal encoding format $\implies$ enable cascading
- Non-linearity of the normalization operation can be used to construct any logic operations
- Very first totally optical logic gate
- Paper in reviewing process

LaTeX

References

[1] P. Ambs. Optical computing: A 60-year adventure. *Hindawi Publishing Corporation*, 2010, 2010.

[2] C. J. B. et al. Laser injection locking. *IEEE*, 61, 1973.

[3] M. S. M. J. Martin-Regalado, F. Prati and N. B. Abraham. Polarization properties of vertical-cavity surface-emitting lasers. *IEEE JOURNAL OF QUANTUM ELECTRONICS*, 33, 1997.

[4] R. A. Morgan. Vertical-cavity surface-emitting lasers: present and future. *SPIE*, 3003, 1997.

[5] V. S. L. A. C. K. H. F. K. T. von Lerber, M. Lassas. Optical computing by injection-locked lasers. 2017.

[6] F. S. Tanenbaum. *Structured Computer Organization*. Pearson, 2013.

Measured BER

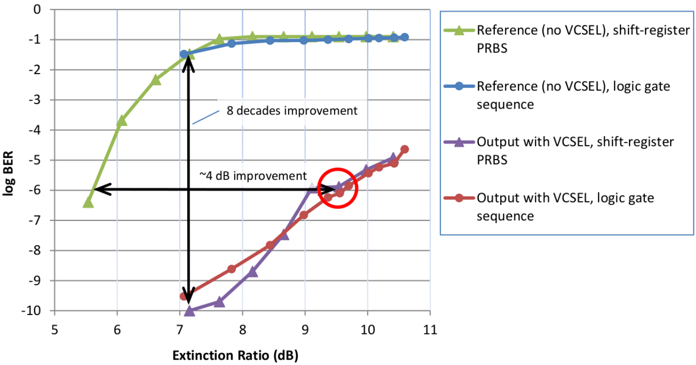

Figure: Measured BER with and without slave VCSEL for varying ER. Red circle: logic gate BER.[5]

VCSEL Sketch

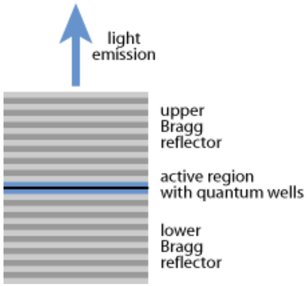

Figure:
www.rp-photonics.com/vertical_cavity_surface_emitting_lasers

Injection-locked laser Sketch

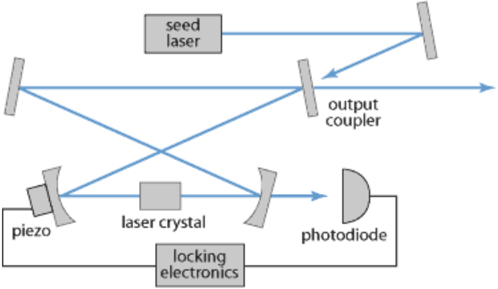

Figure: www.rp-photonics.com/injection_locking

AND

b_0	b_1	b_2	a_0	a_1	a_2	a_{out}	b_{out}	phase
0	0	0	-1	-1	-1	-3	0	π
0	0	1	-1	-1	+1	-1	0	π
0	1	0	-1	+1	-1	-1	0	π
0	1	1	-1	+1	+1	+1	1	0

OR

b_0	b_1	b_2	a_0	a_1	a_2	a_{out}	b_{out}	phase
1	0	0	+1	-1	-1	-1	0	π
1	0	1	+1	-1	+1	+1	0	0
1	1	0	+1	+1	-1	+1	0	0
1	1	1	+1	+1	+1	+3	1	0

- **NOT**: correspond to a phase shift that can be implemented by choosing the length of an optical path that connects two logical units.

All-optical Computer[1]

- Golden age between 1980 and 2004
- Main constraints: speed and loss of SLM (spatial light Modulator)
- Mature areas:
  - communication
  - memory (holographic memory)
- Not mature area:
  - computing
- Now, new start for optical computing by chance of new results from nanooptics, biophotonics or communication system.
- Is the future of optical computing opto-electronic?