Nonlinear Optics

Optics with $\chi^{(3)}$ materials

4.1 Introduction

The main idea is still to use the propagation equation derived from the Maxwell's equation

$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2}{\partial t^2} P_{\mathrm{NL}} \tag{4.1}$$

and which does not depend on the form of the nonlinear polarization P_{NL}. By contrast with non-centrosymmetric crystal all material exhibit third order susceptibility and it is therefore important to study - at least - a few of the nonlinear effects that can result from it.

4.1.1 Nonlinear refractive index

In this case we assume a monochromatic input wave

$$E = \frac{1}{2}\left(\mathcal{E} e^{-i\omega t} + c.c.\right) \tag{4.2}$$

The nonlinear polarization $P_{\mathrm{NL}} = \epsilon_0 \chi^{(3)} E^3$ can be readily calculated

$$P_{\mathrm{NL}} = \frac{\epsilon_0 \chi^{(3)}}{8}\left(\mathcal{E}^3 e^{-3i\omega t} + c.c. + 3\mathcal{E}^* \mathcal{E}^2 e^{-i\omega t} + c.c.\right) \tag{4.3}$$

In this equation, the first term oscillate at 3ω and in a similar way that for $\chi^{(2)}$ materials we could have second harmonic, this term yields the generation of third harmonic (THG). Introduce the complex nonlinear polarisation $\mathcal{P}_{\mathrm{NL}}(t)$ such that

$$P_{\mathrm{NL}} = \frac{1}{2}\left(\mathcal{P}_{\mathrm{NL}}(t) e^{-i\omega t} + c.c.\right) \tag{4.4}$$

we can identify the second term:

$$\mathcal{P}_{\mathrm{NL}} = \frac{3\epsilon_0 \chi^{(3)}}{4} \left|\mathcal{E}(t)\right|^2 \mathcal{E}(t) \tag{4.5}$$

which is a term that oscillate in perfect phase with the input electric field! We should remind that for $\chi^{(2)}$ materials, the polarisation does not oscillate at the frequency of the exciting field and this leads to the notion of *coherence length*. In the present case the situation is drastically different since the polarisation oscillates at the exact same frequency as the incident field. The effects linked with this term are automatically phase-matched and will therefore accumulate along the propagation. We can easily imagine that for the propagation in an optical of a few km this can be problematic. This effect is the *optical Kerr effect*.

Proceeding in the same way that we did for $\chi^{(2)}$ material we can introduce the slowly varying envelope A and express the electric field as $E = A \exp ikz$. Using the nonlinear polarisation that we just calculated we find the evolution of this envelope:

$$\frac{dA}{dz} = \frac{3i\omega\chi^{(3)}}{8nc} |A|^2 A \tag{4.6}$$

We can then get the evolution of the intensity:

$$\begin{aligned} \frac{d|A|^2}{dz} &= A^* \frac{dA}{dz} + \frac{dA}{dz} A^* \\ &= \frac{3i\omega}{8nc} |A|^2 \left(A^* \chi^{(3)} A - A\chi^{(3)*} A^* \right) \\ \Rightarrow \quad \frac{d|A|^2}{dz} &= -\frac{3\omega}{4nc} \operatorname{Im}\left[\chi^{(3)}\right] |A(z)|^4 \end{aligned} \tag{4.7}$$

Depending if the third order susceptibility is real or not this yields different possibilities:

- Non-resonant material - $\chi^{(3)} \in \mathbb{R}$
  In that case the eq. (4.7) is null and this means that $|A(z)|^2 = |A(0)|^2$. The intensity of the beam remains constant over the propagation. We can use this property to calculate the evolution of the envelope using eq. (4.6):

  $$\frac{dA}{dz} = \frac{i3\omega\chi^{(3)}}{8nc} |A(0)|^2 A(z) \tag{4.8}$$

  which is readily integrated to give

  $$A(z) = A(0) \exp\left[\frac{i3\omega\chi^{(3)} |A(0)|^2}{8nc} z\right] \tag{4.9}$$

  This corresponds to an accumulation of a phase term, similar at the one linked with a the refractive index, except this one depends on the intensity of the incident field. We can write the nonlinear phase term $\varphi_{\mathrm{NL}}(z)$ as

  $$\varphi_{\mathrm{NL}}(z) = \frac{\omega}{c} n_2 I z \tag{4.10}$$

  where $n_2 \propto \chi^{(3)}$ is called the *nonlinear refractive index* of the material. Its unit is m^2/W. The general form of the electric field is therefore

  $$\mathcal{A}(z) = \mathcal{A}(0) \exp\left[i\frac{\omega}{c}(n + n_2 I) z\right] \tag{4.11}$$

  which is equivalent to say that refractive index is now intensity-dependent[1] as $n(I) = n + n_2 I$.

- In the vicinity of a resonance the imaginary part $\operatorname{Im}\left[\chi^{(3)}\right] \neq 0$ and therefore the eq. (4.7) can be expressed simpler

  $$\frac{dI}{dz} = -\alpha_2 I^2 \tag{4.12}$$

  where $\alpha_2 \propto \operatorname{Im}\left[\chi^{(3)}\right]$ defines the *two-photon absorption coefficient*. Its origin is indeed different from the linear case since it depends on the square of the intensity of the field.

[1] Remember that in the real world we cannot use plane wave and therefore the intensity of the beam has a certain profile $I(x, y)$. The nonlinear phase-shift is therefore in general spatially dependent $\varphi_{\mathrm{NL}}(x, y, z, I) = ({}^{\omega}/{}_{c})\, n_2 I(x, y) z$. This can lead to autofocalisation of the beam, which can be used for mode-locking laser. We then talk about *Kerr-lens mode-locking*.

4.1.2 Multiple wave mixing

Things become complicated and challenging and more interesting when multiple frequencies interact in the nonlinear material. Let's assume that we have an input consisting of three waves[2] interacting inside our third order material

$$E = E_1(\omega_1) + E_2(\omega_2) + E_3(\omega_3) \tag{4.13a}$$

$$\text{with } E_j(\omega_j) = \frac{1}{2}\left(\mathcal{E}_j(\omega_j, z, t)e^{-i\omega_j t} + c.c\right) \tag{4.13b}$$

In this case the nonlinear polarization is

$$P_{\text{NL}} = P_{\text{NL}}(3\omega) + \sum_{j=1}^{3} P_{\text{NL,j}} \tag{4.14}$$

with

$$P_{\text{NL}}(3\omega) = \frac{\epsilon_0\chi^{(3)}}{8}\sum_{k=1}^{3}\mathcal{E}_k^3 e^{-3i\omega_k t} + c.c. \tag{4.15a}$$

$$\begin{aligned}
P_{\text{NL},1} &= \frac{3\epsilon_0\chi^{(3)}}{8}\left(|\mathcal{E}_1|^2 + 2|\mathcal{E}_2|^2 + 2|\mathcal{E}_3|^2\right)\mathcal{E}_1 e^{-i\omega_1 t} \\
&+ \frac{3\epsilon_0\chi^{(3)}}{8}\left(2|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2 + 2|\mathcal{E}_3|^2\right)\mathcal{E}_2 e^{-i\omega_2 t} \\
&+ \frac{3\epsilon_0\chi^{(3)}}{8}\left(2|\mathcal{E}_1|^2 + 2|\mathcal{E}_2|^2 + |\mathcal{E}_3|^2\right)\mathcal{E}_3 e^{-i\omega_3 t} + c.c
\end{aligned} \tag{4.15b}$$

$$\begin{aligned}
\frac{8}{3}\frac{P_{\text{NL},2}}{\epsilon_0\chi^{(3)}} &= \mathcal{E}_1^2\mathcal{E}_2 e^{-i(2\omega_1+\omega_2)t} + \mathcal{E}_1^2\mathcal{E}_3 e^{-i(2\omega_1+\omega_3)t} \\
&+ \mathcal{E}_1^2\mathcal{E}_2^* e^{-i(2\omega_1-\omega_2)t} + \mathcal{E}_1^2\mathcal{E}_3^* e^{-i(2\omega_1-\omega_3)t} \\
&+ \mathcal{E}_2^2\mathcal{E}_1 e^{-i(2\omega_2+\omega_1)t} + \mathcal{E}_2^2\mathcal{E}_3 e^{-i(2\omega_2+\omega_3)t} \\
&+ \mathcal{E}_2^2\mathcal{E}_1^* e^{-i(2\omega_2-\omega_1)t} + \mathcal{E}_2^2\mathcal{E}_3^* e^{-i(2\omega_2-\omega_3)t} \\
&+ \mathcal{E}_3^2\mathcal{E}_1 e^{-i(2\omega_3+\omega_1)t} + \mathcal{E}_3^2\mathcal{E}_2 e^{-i(2\omega_3+\omega_2)t} \\
&+ \mathcal{E}_3^2\mathcal{E}_1^* e^{-i(2\omega_3-\omega_1)t} + \mathcal{E}_3^2\mathcal{E}_2^* e^{-i(2\omega_3-\omega_2)t} + c.c
\end{aligned} \tag{4.15c}$$

$$\begin{aligned}
\frac{8}{3}\frac{P_{\text{NL},3}}{\epsilon_0\chi^{(3)}} &= 2\mathcal{E}_1\mathcal{E}_2\mathcal{E}_3 e^{-i(\omega_1+\omega_2+\omega_3)t} + 2\mathcal{E}_1\mathcal{E}_2^*\mathcal{E}_3 e^{-i(\omega_1-\omega_2+\omega_3)t} \\
&+ 2\mathcal{E}_1\mathcal{E}_2\mathcal{E}_3^* e^{-i(\omega_1+\omega_2-\omega_3)t} + 2\mathcal{E}_1\mathcal{E}_2^*\mathcal{E}_3^* e^{-i(\omega_1-\omega_2-\omega_3)t} + c.c.
\end{aligned} \tag{4.15d}$$

This is obviously more complicated than in the case of a monochromatic wave. We can however quite straightforwardly identify that the first term (eq. (4.15a)) is a generalization of the third harmonic generation that we already saw in the case of a monochromatic input (eq. (4.3)). On the other hand the second term $P_{\text{NL},1}$, which is the nonlinear modification of the refractive index is much less obvious. Indeed we see that the contribution oscillating at $\exp i\omega_j t$ (with $j = 1, 2, 3$) includes term as $3\left(|\mathcal{E}_j|^2 + 2|\mathcal{E}_k|^2 + 2|\mathcal{E}_l|^2\right)\mathcal{E}_j$. This clearly contrast with the simpler term $3|\mathcal{E}_j|^2\mathcal{E}_j$ that we obtained in the case of a monochromatic input.

These more complicated contributions are actually a manifestation of the interference between two distinct fields $\mathcal{E}_j$ and $\mathcal{E}_{k,l}$. As a general observation these *cross-terms* between

[2] Note that in principle we can have up to 4 waves interacting inside the crystal through a third-order nonlinearity.

fields are affected by a factor 2. As we will see later the term $3\left|\mathcal{E}_j\right|^2\mathcal{E}_j$ leads to the so-called *self-phase modulation* phenomena while the more complicated terms yield *cross-phase* modulation.

The last two terms represent the possible interaction between two and three waves and are direct generalizations of the sum and difference frequencies processes as we saw them in the chapter related to $\chi^{(2)}$ crystals.

4.2 Optical phase-conjugation mirror

4.2.1 Linear case: Fresnel's law

It is well known that when a beam is incident on a surface separating two media (Fig.) it is affected in such a way that part of the beam is reflected and remains in the same medium (refractive index n_1) whilst the another part is refracted into the second medium (refractive index n_2)

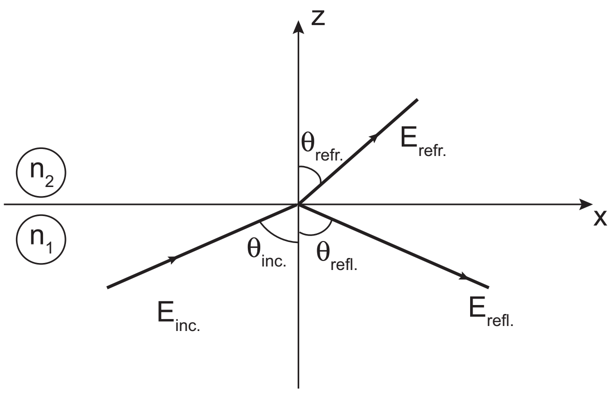

Figure 4.1: Refraction and reflection of a beam at an interface between two media. The refractive indices are $n_1 < n_2$.

The incident field (resp. reflected and refracted ones) can be defined as

$$E_{\text{inc.}} = A\exp\left[i\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right] + c.c. \tag{4.16a}$$

$$E_{\text{refr.}} = A'\exp\left[i\left(\mathbf{k}'\cdot\mathbf{r}-\omega t\right)\right] + c.c. \tag{4.16b}$$

$$E_{\text{refl.}} = A''\exp\left[i\left(\mathbf{k}''\cdot\mathbf{r}-\omega t\right)\right] + c.c. \tag{4.16c}$$

where A is the amplitude of the incident field and $\mathbf{k}$ its wave-vector, which has a norm $k = \left({}^{n\omega}/_{c}\right)$ where ω is the angular frequency of the wave. Considering the boundaries conditions at the interface we can simply write that $k\sin\theta_{\text{inc}} = k'\sin\theta_{\text{refr.}} = k''\sin\theta_{\text{refl.}}$. And since $k \equiv k''$ then $\theta_{\text{inc.}} = \theta_{\text{refl.}}$. The other relation yields the well-known Snell's law

$$\frac{\sin\theta_{\text{inc}}}{\sin\theta_{\text{refr.}}} = \frac{k'}{k} = \frac{n'}{n} \tag{4.17}$$

Although the Snell's laws lead to the direction of the reflected and refracted beam they do not give any indication of the amplitude of the reflected and refracted beam compare to the incident one. That is precisely what the Fresnel coefficients give. Without entering the details of the derivation of these laws[3] we can simply give the main results:

$$A'' = r \cdot A \tag{4.18a}$$
$$A' = t \cdot A \tag{4.18b}$$
$$\tag{4.18c}$$

where the coefficient r and t are respectively the reflection and the transmission coefficient. The conservation of energy imposes that $r^2 + t^2 = 1$.

4.2.2 Nonlinear case

the problem of optical phase conjugation mirror

In the nonlinear case, we replace the traditional mirror by a $\chi^{(3)}$ material. For a very specific set of parameters, which we will establish in the following, this can create a very peculiar mirror where an input field E_{probe} is reflected by the $\chi^{(3)}$ material (Fig. 4.2). More surprising the reflected field is the conjugate of the input one: this is an *optical phase conjugation* setup (OPC).

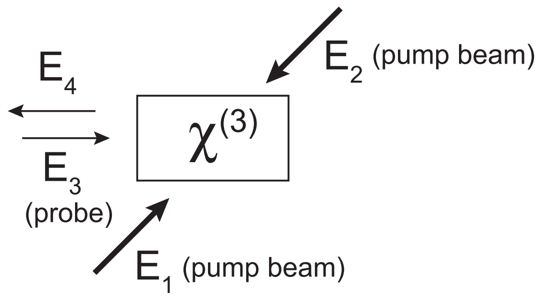

Figure 4.2: Schematic of an optical phase conjugation mirror.

As shown on the Fig. 4.2, to obtain an OPC we need

1. E_1 and E_2 are the pump beam. Their frequencies are ω_1 and ω_2 resp. and the are contra-propagating. Since their wave-vector are resp. $\mathbf{k}_1$ and $\mathbf{k_2}$ we can say that

$$\frac{\mathbf{k}_1}{|\mathbf{k}_1|} + \frac{\mathbf{k}_2}{|\mathbf{k}_2|} = 0 \tag{4.19}$$

   The interaction between these fields create a grating through their interaction in the nonlinear material.
2. The probe beam (E_3) is sent onto the nonlinear material. Its frequency is ω_3 and its wave-vector $\mathbf{k}_3$. It will interact with the grating generated by E_1&E_2 and eventually be reflected.
3. The fourth wave interacting in the crystal is E_4. Since this is – hopefully – the reflection of E_3 the direction of propagation of E_4 is opposite to the one of E_3. Therefore we have

$$\frac{\mathbf{k}_3}{|\mathbf{k}_3|} + \frac{\mathbf{k}_4}{|\mathbf{k}_4|} = 0 \tag{4.20}$$

[3] the coefficient are also dependent of the polarization of the wave, but we do not need to consider this here.

As we already mentioned we will see that E_4 may have very special 1properties such as optical phase conjugation

$$E_3 = A_3 e^{-i(\omega_3 t - \mathbf{k}_3 \cdot \mathbf{r})} + c.c \implies E_4 = A_3^* e^{-i(\omega_4 t - \mathbf{k}_4 \cdot \mathbf{r})} + c.c \tag{4.21}$$

The remaining question is *How to get such property?*

Conditions for optical phase conjugation

As before, we need to calculate the nonlinear polarization $P_{\text{NL}} = \epsilon_0 \chi^{(3)} E^3$ resulting from the interaction between the four fields

$$E = \sum_{j=1}^{4} A_j e^{-i(\omega_j t - \mathbf{k}_j \cdot \mathbf{r})} \tag{4.22}$$

Since we do not want to calculate all the terms from the nonlinear polarization we will only consider the case where

$$\begin{cases} \omega_1 + \omega_2 &= \omega_3 + \omega_4 \\ \mathbf{k}_1 + \mathbf{k}_2 &= \mathbf{k}_3 + \mathbf{k}_4 = 0 \text{ since } E_1 \text{ and } E_2 \text{ have opposite direction} \end{cases} \tag{4.23}$$

The different contribution to the polarization are

$$\frac{P_1}{\epsilon_0 \chi^{(3)}} = 3\left[\left(|A_1|^2 + 2|A_2|^2 + 2|A_3|^2 + 2|A_4|^2\right) A_1 + 2A_2^* A_3 A_4\right] e^{-i(\omega_1 t - \mathbf{k}_1 \cdot \mathbf{r})} \tag{4.24a}$$

$$\frac{P_2}{\epsilon_0 \chi^{(3)}} = 3\left[\left(2|A_1|^2 + |A_2|^2 + 2|A_3|^2 + 2|A_4|^2\right) A_2 + 2A_1^* A_3 A_4\right] e^{-i(\omega_2 t - \mathbf{k}_2 \cdot \mathbf{r})} \tag{4.24b}$$

$$\frac{P_3}{\epsilon_0 \chi^{(3)}} = 3\left[\left(2|A_1|^2 + 2|A_2|^2 + |A_3|^2 + 2|A_4|^2\right) A_3 + 2A_1 A_2 A_4^*\right] e^{-i(\omega_3 t - \mathbf{k}_3 \cdot \mathbf{r})} \tag{4.24c}$$

$$\frac{P_4}{\epsilon_0 \chi^{(3)}} = 3\left[\left(2|A_1|^2 + 2|A_2|^2 + 2|A_3|^2 + |A_4|^2\right) A_4 + 2A_1 A_2 A_3^*\right] e^{-i(\omega_4 t - \mathbf{k}_4 \cdot \mathbf{r})} \tag{4.24d}$$

Now, let us assume that the so-called *pump*-beam E_1 and E_2 are much more intense that the two other fields

$$|E_1|, |E_2| \gg |E_3|, |E_4| \tag{4.25}$$

We can then consider that they are not affected by the presence of E_3 and E_4. Using this assumption we can reduce the eq. (4.24) to

$$\frac{P_1}{\epsilon_0 \chi^{(3)}} = 3\left(|A_1|^2 + 2|A_2|^2\right) A_1 e^{-i(\omega_1 t - \mathbf{k}_1 \cdot \mathbf{r})} \tag{4.26a}$$

$$\frac{P_2}{\epsilon_0 \chi^{(3)}} = 3\left(|2A_1|^2 + |A_2|^2\right) A_2 e^{-i(\omega_2 t - \mathbf{k}_2 \cdot \mathbf{r})} \tag{4.26b}$$

$$\frac{P_3}{\epsilon_0 \chi^{(3)}} = 6\left[\left(|A_1|^2 + |A_2|^2\right) A_3 + A_1 A_2 A_4^*\right] e^{-i(\omega_3 t - \mathbf{k}_3 \cdot \mathbf{r})} \tag{4.26c}$$

$$\frac{P_4}{\epsilon_0 \chi^{(3)}} = 6\left[\left(|A_1|^2 + |A_2|^2\right) A_4 + A_1 A_2 A_3^*\right] e^{-i(\omega_4 t - \mathbf{k}_4 \cdot \mathbf{r})} \tag{4.26d}$$

We now need to use these nonlinear contributions in the propagation equation (eq. (4.1)). Using the slow-varying envelope approximation[4] and since the system is *free-running* we obtain after calculations

$$\frac{dA_1}{dz'} = \frac{3i\omega_1\chi^{(3)}}{2n_1 c}\left(|A_1|^2 + 2|A_2|^2\right)A_1 \tag{4.27a}$$

$$\frac{dA_2}{dz'} = \frac{-3i\omega_2\chi^{(3)}}{2n_2 c}\left(2|A_1|^2 + |A_2|^2\right)A_2 \tag{4.27b}$$

where the $-$sign comes from the fact that we are using direction of propagation of E_1 (z') for the derivation and E_1 and E_2 have opposite direction. Note that as we noticed in the case of second harmonic we have here

$$\frac{d|A_1|^2}{dz'} = \frac{d|A_2|^2}{dz'} = 0 \tag{4.28}$$

meaning that the pump beams E_1 and E_2 are not affected during the process. This also means that the pump beams simply acquire a phase-shift during their propagation in the third-order material

$$A_1(z') = A_1(0)e^{i\gamma_1\left(|A_1|^2+2|A_2|^2\right)z'} \tag{4.29a}$$

$$A_2(z') = A_2(0)e^{-i\gamma_2\left(|2A_1|^2+|A_2|^2\right)z'} \tag{4.29b}$$

where $\gamma_i = \left(3\omega_j\chi^{(3)}/2n_j\epsilon_0 c\right)$. Note that in the particular case where $\gamma_1\left(|A_1|^2 + 2|A_2|^2\right) = \gamma_2\left(2|A_1|^2 + |A_2|^2\right)$, which is equivalent as

$$\begin{aligned}
\left(\frac{3\omega_1\chi^{(3)}}{2n_1\epsilon_0 c}\right)\left(|A_1|^2 + 2|A_2|^2\right) &= \left(\frac{3\omega_2\chi^{(3)}}{2n_2\epsilon_0 c}\right)\left(2|A_1|^2 + |A_2|^2\right)\\
\Leftrightarrow \quad n_2\omega_1\left(|A_1|^2 + 2|A_2|^2\right) &= n_1\omega_2\left(2|A_1|^2 + |A_2|^2\right)\\
\Leftrightarrow \quad \frac{|A_1|^2}{|A_2|^2} &= \frac{2n_2\omega_1 - n_1\omega_2}{2n_1\omega_2 - n_2\omega_2}
\end{aligned} \tag{4.30}$$

then the product of the field is space-invariant:

$$A_1(z')A_2(z') = A_1(0)A_2(0) \tag{4.31}$$

Similarly the coupled equations for the probe beam are

$$\frac{dA_3}{dz} = \frac{3i\omega_3\chi^{(3)}}{n_3 c}\left[\left(|A_1|^2 + |A_2|^2\right)A_3 + A_1A_2A_4^*\right] \tag{4.32a}$$

$$\frac{dA_4}{dz} = \frac{-3i\omega_4\chi^{(3)}}{n_4 c}\left[\left(|A_1|^2 + |A_2|^2\right)A_4 + A_1A_2A_3^*\right] \tag{4.32b}$$

---

[4] We use the same procedure as for the second harmonic process where we used the slow-varying envelope approximation to derive the equation (2.21)

$$e^{ik_m z}\left(2ik_m\partial_z A_m + \frac{2in_m^2\omega_m}{c^2}\partial t A_m\right) = -\frac{\omega_m^2}{\epsilon_0 c^2}P_{\mathrm{NL}}$$

where we insert the appropriate polarization term eq. (4.26a) in order to get

$$\partial z' A_1 + \frac{n_1}{c}\partial_t A_1 = \frac{3i\omega_1\chi^{(3)}}{2n_1 c}\left(|A_1|^2 + 2|A_2|^2\right)A_1$$

And in the free running case the time-derivative drops, yielding the eq. (4.27a).

In the particular case where the assumption (eq. (4.30)) is fulfilled then the coefficients of $(|A_1|^2 + |A_2|^2)$ as well as $A_1 A_2$ are space-invariant and therefore the coupled equations (eq. (4.32) are linear). Additionally in the case where $\omega_3/n_3 = \omega_4/n_4$ then these equations (eq. (4.32)) have a phase conjugation solution

$$A_4(z) = A_3^*(z) \tag{4.33}$$

If we have $\omega_3/n_3 \neq \omega_4/n_4$ then the general solution of the coupled equations (eq. (4.32)) is

$$A_4(z) = \frac{n_3\omega_4}{n_4\omega_4} A_3^*(z) \tag{4.34}$$

This is quite surprising since the *reflection coefficient* $r = (n_3\omega_4/n_4\omega_3)$ can be larger than unity. Not only the input beam is reflected and possesses the properties of phase-conjugation but it can also be amplified!

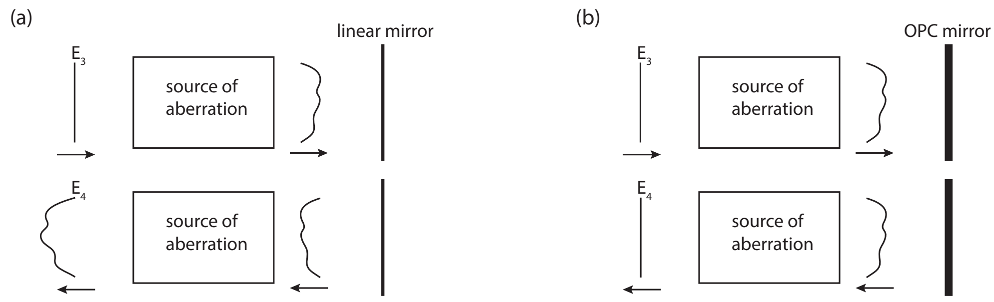

Figure 4.3: Comparison between (a) a linear mirror and (b) a optical phase conjugation mirror for compensation of phase aberration.

Obviously these results require many specific conditions but it contrasts so much with a traditional mirror that it is an important application of third-order nonlinearity. If we remind that in the case of a linear mirror we had $A_4(z) = rA_3(z)$ with $r < 1$ we can clearly see that the situation is totally different here. In particular since the pump beam are contra-propagating ($\mathbf{k}_1 + \mathbf{k}_2 = 0$) then the wave-vector of the reflected beam $\mathbf{k}_4$ has to fulfill ($\mathbf{k}_3 + \mathbf{k}_4 = 0$). This means $(k_{4x}, k_{4y}, k_{4z}) = (-k_{3x}, -k_{3y}, -k_{3z})$. All three components[5] are reversed! It is particularly important to balance the effect of phase aberration, which could yield modification of the wavefront of a beam (Fig. 4.3).

[5]This is obviously in strong contrast with the case of a linear mirror where we have $(k_{4x}, k_{4y}, k_{4z}) = (k_{3x}, k_{3y}, -k_{3z})$.

Bibliography

[1] G. Stegeman and R. Stegeman, *Nonlinear Optics*, U. o. C. F. Glenn Boreman, Ed. John Wiley & Sons, Inc., 2012.

[2] P. Mandel, *Nonlinear Optics*. Wiley-VCH Verlag GmbH, 2010.

[3] R. W. Boyd, *Nonlinear Optics, Third Edition*, 3rd. Orlando, FL, USA: Academic Press, Inc., 2008, ISBN: 0123694701, 9780123694706.

[4] G. New, *Introduction to Nonlinear Optics*. Cambridge University Press, 2011.