Modern Optics: Advanced optics
Fraunhofer diffraction

Exercises' sheet No 5 Jan. 2018

Exercise 1 *Cylindrical waveguide*

During the lecture we saw that the eigenvalue equation for cylindrical step-index waveguide could be expressed in the form :

$$\left[\frac{J_n'(U)}{UJ_n(U)}+\frac{K_n'(W)}{WK_n(U)}\right]\left[n_{cl}^2\frac{J_n'(U)}{UJ_n(U)}+n_{co}^2\frac{K_n'(W)}{WK_n(U)}\right] = \frac{\beta^2}{k^2}\left(\frac{1}{U^2}+\frac{1}{W^2}\right)n^2 = \left(\frac{n_{co}^2}{U^2}+\frac{n_{cl}^2}{W^2}\right)n^2 \tag{1}$$

which can be solved numerically. This consists of finding the zeros of the function (Fig. 1). Unfortunately the resolution of this equation is rather difficult since two consecutive zeros (*i.e.* for EH_{11} and for HE_{12}) are very close to each other. Another important point is that we distinguish the HE-modes from the EH-modes in a rather arbitrary way. The goal here is to uncover this ambiguity.

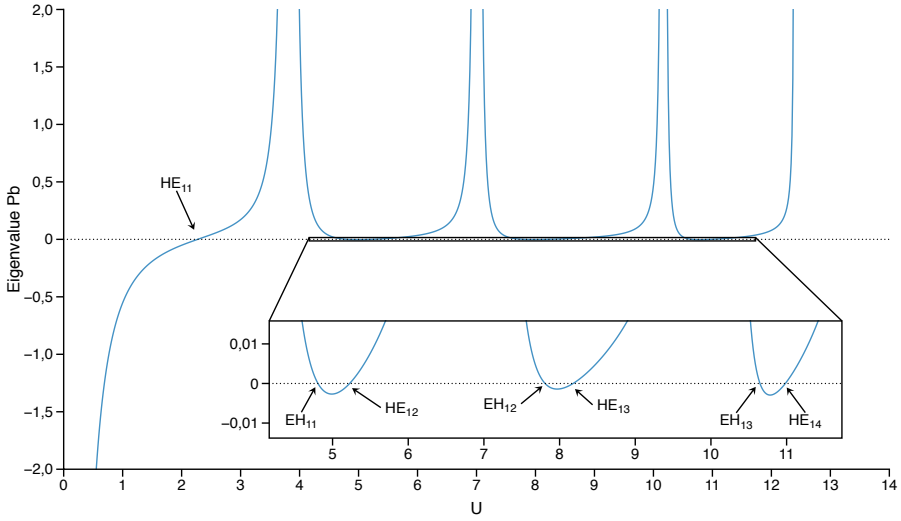

Figure 1 – Eigenvalue problem eq. (1) for a fibre with a diameter $2a = 4\mu$m. The wavelength is 1064 nm. This corresponds to $V = 12.395$ for the considered waveguide (silica core surrounding by air). The arrow indicates the zeros and the corresponding optical modes.

1. Write the eq. (1) by using the change of variable

$$x = \frac{J_n'(U)}{UJ_n(U} \quad K = \frac{K_n'(W)}{WK_n(W)} \quad C = \frac{n^2\beta^2}{k^2}\left(\frac{1}{U^2}+\frac{1}{W^2}\right)^2 \tag{2}$$

2. Using the recurrence relation for the Bessel functions

$$J_n'(x) = -J_{n+1}(x)+\frac{n}{x}J_n(x) = J_{n-1}(x)-\frac{n}{x}J_n(x) \tag{3}$$

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show that the solutions of eq.(2) can be written as two new eigenvalue equations :

$$\frac{J_{n+1}(U)}{UJ_n(U)} = \left(\frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2}\right)\frac{K_n'(W)}{WK_n(W)} + \left(\frac{n}{U^2} - R\right) \tag{4a}$$

$$\frac{J_{n-1}(U)}{UJ_n(U)} = -\left(\frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2}\right)\frac{K_n'(W)}{WK_n(W)} + \left(\frac{n}{U^2} - R\right) \tag{4b}$$

where R must be expressed.

3. The eq. (4a) actually corresponds to the EH-mode and the eq. (4b) to the HE-mode. By plotting these equations, show that there can be situation where the waveguide will not support any EH-mode whilst at least one HE-mode is always supported.
4. How can we write these equations for weakly guiding optical fiber ?