Advanced laser
The Z-cavity - Titanium-sapphire laser

0.1 Introduction

One of the most used laser for ultrashort pulse is certainly the Titanium:Sapphire laser (Ti:Sa), and it relies on Kerr-lens mode-locking (KLM). For KLM, it is important to tightly focus the laser beam inside the nonlinear crystal in order to enhance the optical Kerr effect $n = n_0 + n_2 I$, inside the crystal. Pulses can then create an instantaneous lens inside the crystal, while CW regime does not modify the cavity. The goal here is to study the particular configuration of the cavity of Ti:Sa in order to see the subtle tricks that are necessary to achieve good performance of the laser. At first, we will focus on the empty cavity (without crystal nor prisms) and see that the angle of the folding spherical mirrors is very critical. Astigmatism will then be discussed. Finally, We will include the Kerr-lens inside the cavity.

A typical cavity for mode-locked Ti:Sa is shown on figure 1. Crystal are typical 30 mm long for commercial systems, but other lengths are also possible.

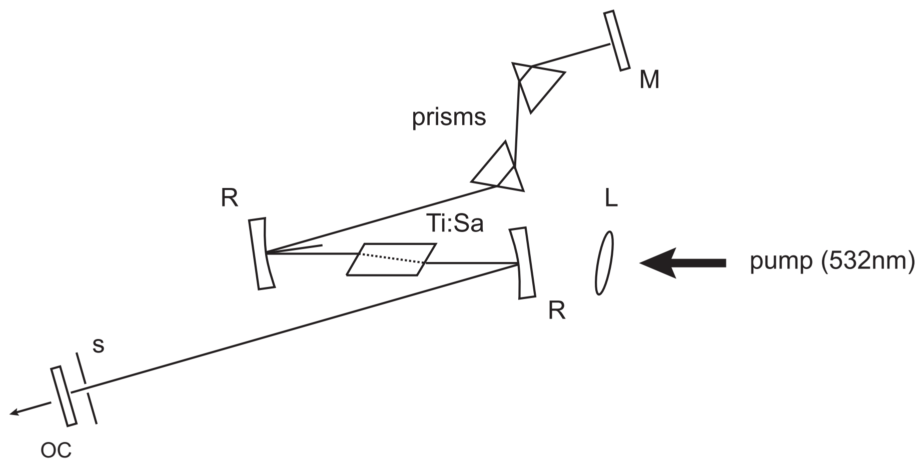


Figure 1: Schematic of the typical Z-cavity of a Ti:Sa mode-locked laser. OC is the output coupler, S a slit, R two spherical mirrors, and L the lens used to couple the optical pump inside the Ti:Sa crystal.

As we studied during the *basics of Lasers* lecture, we could use the ABCD matrices in order to calculate the stability of such a cavity, and compute the evolution of the beam waist inside this cavity. We will first see that this complicated cavity can reduce to a simple spherical-spherical Fabry-Perot cavity, much easier to deal with.

Equivalent cavity

First of all, we will show that the combination of a single lens with a flat mirror can be seen as a simple spherical mirror

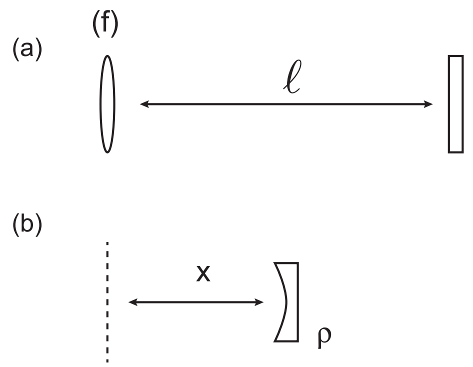


Figure 2: Equivalence between a single spherical mirror and a system composed as a lens and a flat mirror.

The ABCD matrix for each situation can easily be calculated:

$$M_{\text{lens+flatmirror}} = \begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2\ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{2\ell}{f} & 2\ell \\ \frac{-2}{f} + \frac{2\ell}{f^2} & 1 - \frac{2\ell}{f} \end{bmatrix} \tag{1a}$$

$$M_{\text{spher.mirror}} = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{2}{\rho} & 1 \end{bmatrix} \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{2x}{\rho} & 2x - \frac{2x^2}{\rho} \\ -\frac{2}{\rho} & 1 - \frac{2x}{\rho} \end{bmatrix} \tag{1b}$$

Both matrices are equivalent, if and only if every components are equal. After simple linear algebra, we find that this is indeed correct if and only if

$$\rho = \frac{f^2}{f - \ell} \tag{2a}$$

$$x = \frac{f\ell}{f - \ell} \tag{2b}$$

Finally we can draw the Z-cavity laser

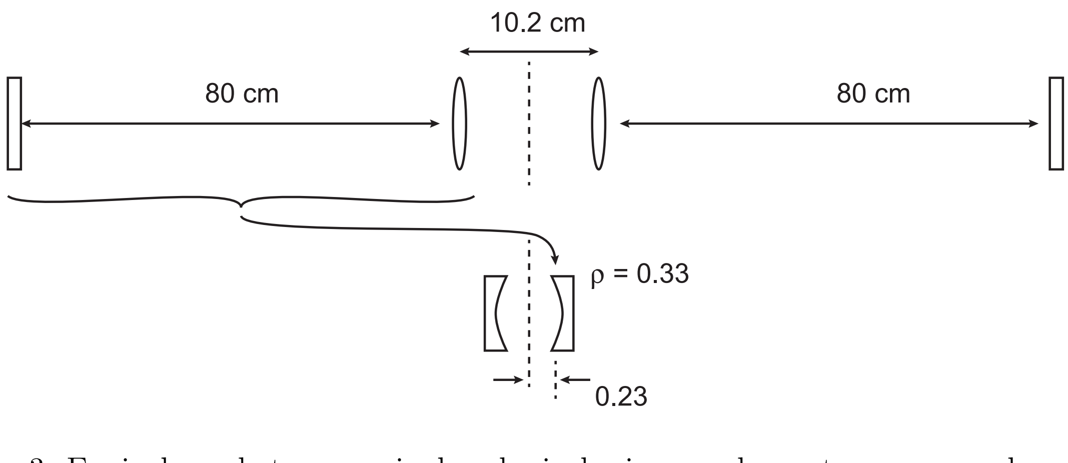


Figure 3: Equivalence between a single spherical mirror and a system composed as a lens and a flat mirror.

From this equivalent cavity, we can easily check its stability:

$$g_1 = g_2 = \left(1 - \frac{L}{\rho}\right) = \left(1 - \frac{0.46}{0.33}\right) \simeq -0.39$$
$$\Rightarrow |g_1 g_2| \simeq 0.155 \leq 1$$

The cavity is stable.

astigmatism

Now that we know that the complicated cavity is equivalent to a simple stable spherical-spherical cavity, we could simply calculate the size of the beam in the crystal and the problem would be over. Actually, situation is slightly more complex since the folding mirror used in the Z-cavity are not used at normal incidence. This leads to *astigmatism* and a spatial deformation of the laser beam: the beam has no longer a cylindrical symmetry! The fig. 4 shows the definition of the *sagittal* and *tangential* planes for a simple lens.

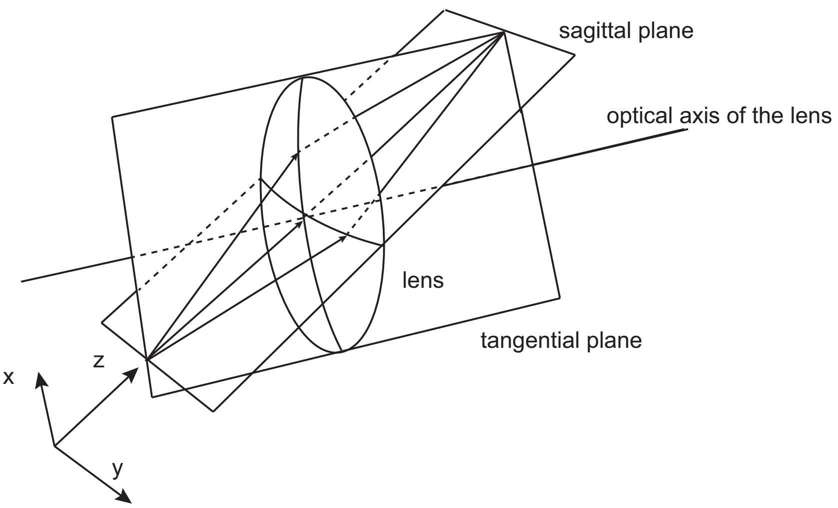


Figure 4: definition of the sagittal and tangential planes for a lens. The beam propagates along the (Oz) axis, which does not coincide with the optical axis of the lens.

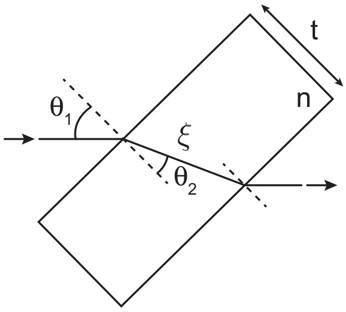
Depending on the considered plane, the ABCD matrix for a mirror (radius of curvature R) is now given as

$$M_{\text{sag}} = \begin{bmatrix} 1 & 0 \\ \dfrac{-2\cos\theta}{R} & 1 \end{bmatrix} \tag{3a}$$
$$M_{\text{tgt}} = \begin{bmatrix} 1 & 0 \\ \dfrac{-2}{R\cos\theta} & 1 \end{bmatrix} \tag{3b}$$

Similarly for a tilted interface, the transfer matrix depends on the considered plane:

$$M_{\text{sag}} = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1}{n} \end{bmatrix} \tag{4a}$$
$$M_{\text{tgt}} = \begin{bmatrix} \dfrac{\cos\theta_2}{\cos\theta_1} & 0 \\ 0 & \dfrac{\cos\theta_1}{n\cos\theta_2} \end{bmatrix} \tag{4b}$$

Therefore the Ti:Sa crystal must be modelled by two distinct transfer matrices, depending of the considered planes:



$$M_{\text{sag}} = \begin{bmatrix} 1 & \dfrac{\xi}{n} \\ 0 & 1 \end{bmatrix} \tag{5a}$$

$$M_{\text{tgt}} = \begin{bmatrix} 1 & \dfrac{\xi \cos^2 \theta_1}{n \cos^2 \theta_2} \\ 0 & 1 \end{bmatrix} \tag{5b}$$

where $t = \xi \cos \theta_2$. Moreover, in order to minimize the loss inside the cavity, the crystal is cut at the Brewster angle. Therefore the incident angle θ_1 is such that $\tan \theta_1 = n_0$. The distance inside the crystal ξ can then be expressed as a function of the physical thickness of the crystal t and its refractive index n[1]:

$$\xi = \frac{t\sqrt{n^2+1}}{n} \tag{7}$$

and then the transfer matrices for the crystal:

$$M_{\text{sag}} = \begin{bmatrix} 1 & \dfrac{\xi}{n} \\ 0 & 1 \end{bmatrix} \tag{8a}$$

$$M_{\text{tgt}} = \begin{bmatrix} 1 & \dfrac{\xi}{n^3} \\ 0 & 1 \end{bmatrix} \tag{8b}$$

It is now time to merge all these results together in order to find the real length of the equivalent cavity depending of the considered planes:

On Fig. 5, $\mathscr{L}$ is the length of the equivalent cavity

$$\mathscr{L} = (\ell - t_{\text{crystal}}) + 2x + \ell_{\text{opt.}} \tag{9}$$

with ℓ the physical distance between the folding mirror in the Z-cavity, t_{crystal} the thickness of the crystal, x the additional distance (eq. (2b)) and ℓ_{opt} the optical path within the crystal. Using eq. (2), and since the focal of the mirror is $f = \left({}^{R}\!/\!_{2}\right)$, we can notice that

$$\begin{aligned} 2x &= \frac{2f\ell}{f-\ell} = \frac{2f^2 - 2f^2 + 2f\ell}{f-\ell} \\ &= 2\frac{f^2}{f-\ell} - 2f \\ \rightarrow\ 2x &= 2\rho - R \end{aligned} \tag{10}$$

[1]

$$\begin{aligned} \xi &= \frac{t}{\cos\theta_r} = \frac{t}{\sin\theta_i} = \frac{t\ \cos\theta_i}{\sin\theta_i \cos\theta_i} = \frac{t\sqrt{\dfrac{\cos^2\theta_i + \sin^2\theta_i}{\cos^2\theta_i}}}{\tan\theta_i} \\ &= \frac{t\ \sqrt{1+\tan^2\theta_i}}{\tan\theta_i} = \frac{t\ \sqrt{1+n^2}}{n} \end{aligned} \tag{6}$$

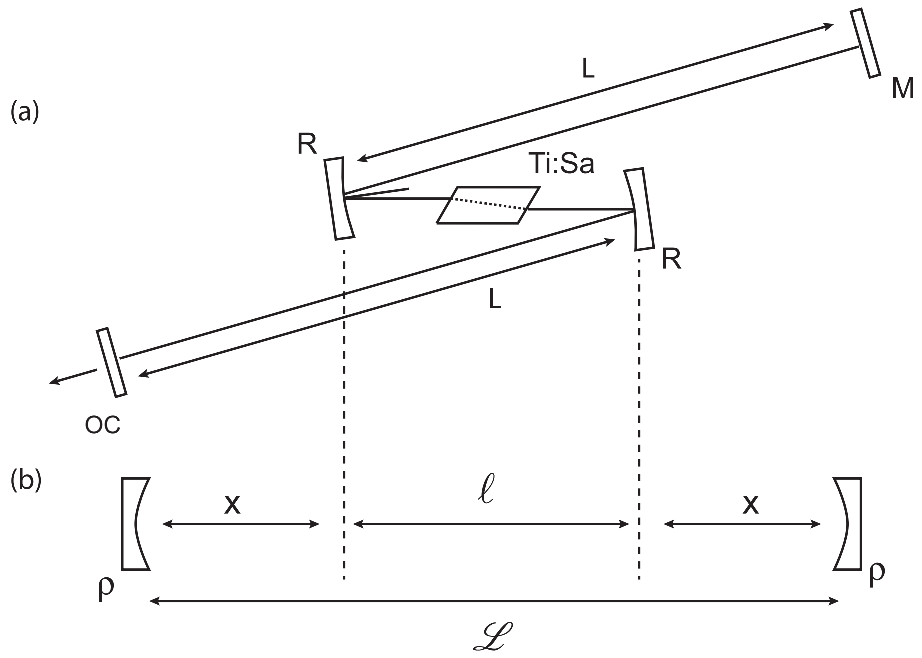


Figure 5: Z-cavity and its equivalent as a spherical-spherical cavity

Therefore eq. (9) can also be written

$$\mathscr{L}_i = (\ell - t_{\text{crystal}}) + (2\rho_i - R_i) + \ell_{\text{opt.}} \tag{11}$$

where the subscript i indicates the sagittal or the tangential plane. Considering the radius of curvature of the equivalent mirror the effective length of the equivalent cavity (eq. (11)) is then

$$\mathscr{L}_{\text{sag}} = (\ell - t) + \frac{\xi}{n} + 2\frac{f^2/\cos^2\theta}{f/\cos\theta - L} - \frac{R}{\cos\theta} \tag{12a}$$

$$\mathscr{L}_{\text{tgt}} = (\ell - t) + \frac{\xi}{n^3} + 2\frac{f^2\cos^2\theta}{f\cos\theta - L} - R\cos\theta \tag{12b}$$

The only parameter here is the angle of the folding mirrors (θ). We can plot the evolution of the waist at the center of the equivalent cavity. Of course, it is important to plot this quantity for both sagittal and tangential planes since the cavity will only be stable at the overlap between these stability region (fig. 6).

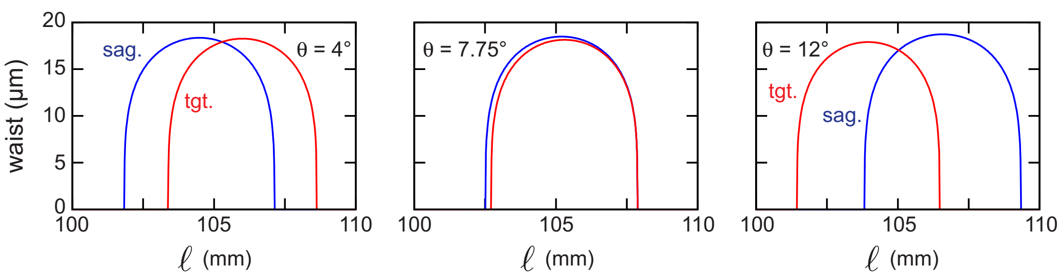


Figure 6: evolution of the waist as a function of the real distance between the folding mirror in the Z-cavity. The Ti:Sa crystal is t=4 mm long and n=1.76.

It is clear from the fig. 6 that the angle of the folding mirror is quite critical. A wrong angle will result in a reduction of the stability region of the laser. Of course, we focused our lecture on the particular case of the Z-cavity of the mode-locked Ti:Sa laser system, but these calculations will apply to any Z-cavity, no matter whether it is dedicated to mode-locking or not.

Kerr-lens mode-locking

The means of mode-locking the Ti:Sa laser is the efficient use of a self-induced lens, created by the traveling pulse itself when it passes through the crystal. Fig. 7 shows how the Kerr effect inside the crystal will modify locally the laser beam. This corresponds to the instantaneous (response time is on the fs-scale) creation of a lens.

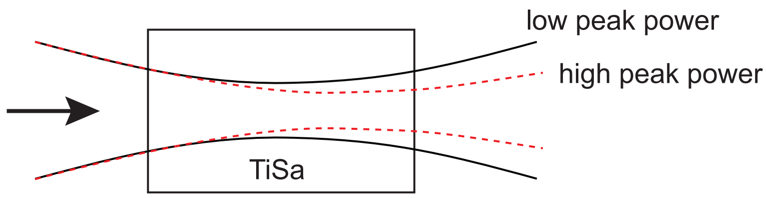


Figure 7: Influence of the instantaneous Kerr-lens on the spatial characteristics of the laser beam inside the Ti:Sa crystal.

On Fig. 8, we plotted the evolution of the beam waist, calculated in the center of the equivalent cavity calculated at the beginning of this chapter (Fig. 3). As expected, the equivalent cavity is stable for a length[2] $\ell \in [0; 6.6[$ mm, since $\rho = 3.3$ mm. However, in the presence of an extra lens, a hole in the domain of stability appear. All the calculations for both sagittal and tangential planes could be done again in order to see how the Kerr-lens influences both domain of stability. In practice, the system works if the lens induced a significant change of the spatial properties of the beam. An slit inserted inside the cavity (Fig. 1) can induce additional losses on the non-pulsed regime, while leaving the pulsed regime evolving without any perturbation. Note that this slit will not only allow generating pulses, but it will also prevent background (CW component) to remain in the system.

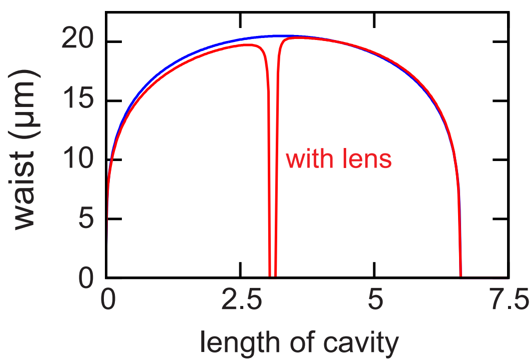


Figure 8: Evolution of the beam waist in the equivalent spherical-spherical cavity of fig.3, but with a convergent lens located inside the cavity. The focal of the lens is 15 mm, and is located at 250 μm from the center of the cavity. We only considered the sagital plane here.

[2]Here ℓ is the distance between the mirror in the equivalent cavity.