Modern Optics II: Nonlinear Optics
Sheet VIII

Nonlinear Schrödinger equation July 2017

Exercise 1 *Modulation instability*

In the case of CW pumping inside a fibre, the propagation equation is simply given by the nonlinear Schrödinger equation :

$$i\partial_z A = \frac{1}{2}\beta_2 \partial_{tt} A - \gamma |A|^2 A \tag{1}$$

Since the pumping is CW, solution of eq. (1) is simply given as

$$A = \sqrt{P_0} \exp(i\gamma P_0 z) \tag{2}$$

1. check the stability of this soliton (eq. (2)) against small perturbation and find the location of the modulation instability frequencies.

Exercise 2 *Four-wave mixing*

FWM can be cascaded, when newly generated signals are further mixed with the existing pump. If we propagate two pumps, ω_p & ω_{-p}, separated by Δ and symetrically centered around ω_0 such that $\omega_{\pm p} = \omega_0 \pm {}^{\Delta}/_{2}$, a frequency comb of discrete sidebands separated by Δ will emerge. The frequency of the nth component (n = 1, 2, 3, ...) is $\omega_{\pm n} = \omega_{\pm p} \pm n\Delta$ where the subscripts + and - indicate frequency up- or down-shifting. The nth component signal of the cascade will be coherently amplified when the phase of the pump $\varphi(z)$ and the phase of the signal φ_n are matched throughout propagation along z, that is,

$$\varphi_{\pm n}(z) = (n+1)\varphi_{\pm p}(z) - n\varphi_{\mp p}(z) \tag{1}$$

Determine the frequency at which phase-mathing occurs if self-phase mod- ulation and cross-phase modulation can be ignored. (Hint : Determine the linear phase evolution along propagation $\varphi(z)$ for both pump and signal, and use Taylor expansion to expand any $\beta(\omega)$ you encounter around ω_0)

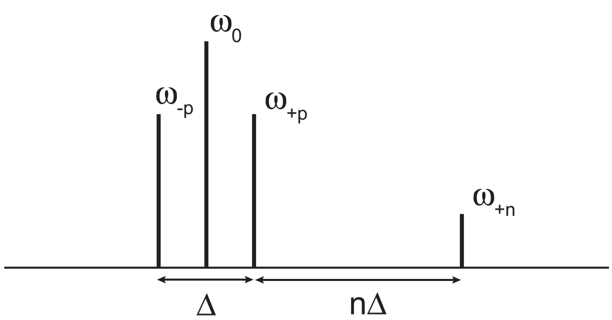

Figure 1 – Illustration of cascaded four wave mixing