Modern Optics II: Nonlinear Optics
Sheet VII

Nonlinear Schrödinger equation July 2017

Exercise 1 *Pulse propagation*

A pulse propagating in a fibre is temporally broadened as a result of the fibre's dispersion. This effect can be avoided by use of dispersion compensation, for example by using fibres with opposite dispersion sign.

1. Given 1 ps pulse with central wavelength $\lambda_0 = 800$ nm propagating in an optical fibre made of silica ($\beta_2 = 200$ ps^2/km) calculate the duration of the pulse after 1 m propagation in absence of nonlinearity.
2. Repeat the calculation for a 100 fs pulse.
3. Adding a second fibre with negative dispersion the pulse can be recompressed to the initial duration : find a combination of negative dispersion and fibre length which exactly compensate the chirp introduced by the first fibre.
4. What's the required input power to compress a 50 fs pulse down to 10 fs, if a fibre with $\gamma = 1$ W^{-1}m^{-1} and negative dispersion is used.

Exercise 2 *Solitons and dispersive waves*

The nonlinear Schrödinger equation (NLSE) is

$$i\partial_z A = \frac{1}{2}\beta_2 \partial_{tt} A - \gamma |A|^2 A \tag{1}$$

1. Check that depending of the sign of β, the equation (1) accept two types of pulse as solution :

$$\beta = -1 \rightarrow A_0 \ \mathrm{sech}(\tau) \exp\left(\mathrm{i}\frac{\gamma|\mathrm{A}_0|^2}{2}\mathrm{z}\right) \quad \textit{: bright soliton}$$
$$\beta = +1 \rightarrow A_0 \ \tanh(\tau) \exp\left(\mathrm{i}\frac{\gamma|\mathrm{A}_0|^2}{2}\mathrm{z}\right) \quad \textit{: dark soliton}$$

2. What is the dispersion relation of the soliton of a bright soliton ?
3. Write the dispersion relation for a linear wave. Conclusion ?
4. What happens when the third order dispersion β_3 is now included in the eq. (1) ? Modify the dimension of the equation in order to introduce the parameter $\delta = \beta_3/(6\beta_2)$.

Note : We remind that

$$\frac{d \ \mathrm{sech(x)}}{dx} = -\mathrm{sech(x)} \ \ \mathrm{tanh(x)} = -\frac{\mathrm{sinh(x)}}{\mathrm{cosh(x)}}$$
$$\frac{d \ \mathrm{tanh(x)}}{dx} = 1 - \tanh^2(x)$$