Modern Optics II: Nonlinear Optics
Sheet III

$\chi^{(2)}$-processes May 2018

Exercise 1 *Invariant in Second Harmonic Generation*

For perfect phase-matching, the coupled equation for the second harmonic generation :

$$\frac{d\rho_1}{d\zeta} = \rho_1\rho_2 \sin\theta \tag{1a}$$

$$\frac{d\rho_2}{d\zeta} = -\rho_1^2 \sin\theta \tag{1b}$$

$$\frac{d\theta}{d\zeta} = \left(2\rho_2 - \frac{\rho_1^2}{\rho_2}\right)\cos\theta + \ell\Delta k \tag{1c}$$

lead to the invariant :

$$\rho_1^2\rho_2 \cos\theta = G = \text{cste} \tag{2}$$

To demonstrate this, we started from the eq. (1c), written as

$$\frac{d\theta}{d\zeta} = \frac{\cos\theta}{\sin\theta}\frac{d}{d\zeta}\left[\ln\left(\rho_1^2\rho_2\right)\right] + \ell\Delta k \tag{3}$$

1. Demonstrate the equation (3).
2. Find the invariant in the case of imperfect phase-matching $\Delta k \neq 0$

Exercise 2

We want to realise the second harmonic of a 1064 nm laser using a KDP nonlinear crystal. The nonlinear coefficient is $d_{\text{eff}} = 4.35 \times 10^{-13}$ m/W. The refractive indices at the fundamental and the second harmonic are respectively $n(1064) = 1.4938$ and $n(532) = 1.5123$. We focus a 100 fs pulse laser (1 nJ) energy into the 3 mm long crystal. We assume that the beam is 30 μm diameter at the focus point at the fundamental ($\lambda = 1064$ nm). We suppose that we have perfect phase-matching.

1. Show that in the case of weak depletion of the pump, the intensity of the second harmonic is given by

$$I_2(z) = \frac{2\left(\omega_1\chi^{(2)}\right)^2}{n_1^2 n_2 c^3 \epsilon_0} z^2 I_1^2 \,\text{sinc}^2\left(\frac{z\cdot\Delta k}{2}\right) \tag{1}$$

2. Calculate the intensity of the second harmonic beam at the output of the crystal.
3. Is the assumption of weak depletion of the pump valid ?

Exercise 3

Let's consider a crystal, that presents a 3^{rd}-order electric susceptibility such that the induced atomic polarisation is $P_{NL} = \epsilon_0\chi^{(3)}E^3$. For an input field

$$\mathbf{E} = E_1 e^{i\omega_1 t} + E_2 e^{i\omega_2 t} + E_3 e^{i\omega_3 t} + c.c.$$

- From symmetry considerations, find the amplitudes of the contributions at ω_1, ω_2, ω_3, $3\omega_1$, $3\omega_2$,$3\omega_3$, $(\omega_1 + \omega_2 + \omega_3)$, $(\omega_1 + \omega_2 - \omega_3)$, $(\omega_1 - \omega_2 + \omega_3)$, $(-\omega_1 + \omega_2 + \omega_3)$, $(2\omega_1 \pm \omega_2)$, $(2\omega_1 \pm \omega_3)$.
  note : you may start with a χ^2 crystal, and $E = E_1 e^{i\omega_1 t} + E_2 e^{i\omega_2 t} + c.c.$.
- Check that $P(2\omega_i)$ and $P(0)$ are null.