Modern Optics II: Nonlinear Optics
Second harmonic generation

Exercises' sheet No 1bis April 2018

Exercise 1 *the Franken's experiment*

In the lecture we discuss the first experiment ever published on second harmonic and the fact that the conversion efficiency was so small that the result does not actually appear in the paper. The question here is to find out why it was so.

1. Considering an electric field incoming on a non centro-symmetric crystal

$$\frac{1}{2}\left[\mathcal{E}_1 e^{-i(\omega_1 t - k_1 z)} + c.c.\right] \tag{1}$$

   where k_1 is the wavenumber. Write the induced polarisation.

2. Considering that the induced polarisation oscillating at $2\omega_1$ will lead to the generation of a wave at that second harmonic frequency, express the field $E_2(2\omega)$.
3. By comparing the two expressions, what can you conclude ? How would you define the coherence length ?

Exercise 2 *Amplitude equations*

Let's consider two electric fields incoming on a second-order nonlinear crystal E_1 and E_2 such that E_2 is the second harmonic of E_1 :

$$E = \frac{1}{2}\left[\mathcal{E}_1 e^{-i\omega t} + c.c + \mathcal{E}_2 e^{-2i\omega t} + c.c\right] \tag{1}$$

1. Write the optically induce polarisation oscillating at ω and at 2ω.
2. Use the derived polarisation as driving terms for the propagation equation for both E_1 and E_2.
3. Show that by introducing the spatial dependence such that $E_m = A_m e^{ik_m z}$ with k_m the wavenumber at the considered frequency ω_m you can derive amplitude equation for both electric fields :

$$\partial_z A_1 + \frac{n_1}{c}\partial_t A_1 = \frac{i\omega_1\chi^{(2)}}{n_1 c} A_1^* A_2 e^{i(k_2 - 2k_1)z} \tag{2a}$$

$$\partial_z A_2 + \frac{n_2}{c}\partial_t A_2 = \frac{i\omega_2\chi^{(2)}}{2n_2 c} A_1^2 e^{-i(k_2 - 2k_1)z} \tag{2b}$$

   Hint : you have to use the slow-varying approximation.

4. Show that in the free running case the total energy is conserved.

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