
Nonlinear Optics

ELECTRO-OPTICAL EFFECTS

3.1 Electro-optical effects

3.1.1 Tensors and index ellipsoid

We have already discussed that since \mathbf{D} and \mathbf{E} are linked by a tensorial relationship $\mathbf{D} = \epsilon_0 \hat{\epsilon}_r \mathbf{E}$, where $\hat{\epsilon}_r$ is a tensor. We can rewrite this equation as

$$\forall i \in (x, y, z), \quad D_i = \epsilon_0 \sum_j \hat{\epsilon}_{r,ij} E_j \quad (3.1)$$

In the most general form the tensor $\hat{\epsilon}_r$ is

$$\hat{\epsilon}_r = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} \epsilon_1 & \cdots & \\ \vdots & \epsilon_2 & \\ & & \epsilon_3 \end{bmatrix} \quad (3.2)$$

where ϵ_1 , ϵ_2 and ϵ_3 are the susceptibility along the three eigenaxis of the tensor $\hat{\epsilon}_r$. We remind that we previously wrote from the tensor $\hat{\epsilon}_r$ the index ellipsoid

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (3.3)$$

where $n_i = \sqrt{\epsilon_i}$. We also remind that depending on the values of the refractive indices n_i , the material can be characterised as

- *isotropic material* if $\epsilon_1 = \epsilon_2 = \epsilon_3$
- *uniaxial material* if $\epsilon_1 = \epsilon_2 \neq \epsilon_3$
- *biaxial material* if $\epsilon_1 \neq \epsilon_2 \neq \epsilon_3$

3.1.2 Impermeability

In the case of electro-optical material the refractive of index can be modified when an external field (DC or not) is applied. Actually, such a modification of the refractive index will affect the index ellipsoid, but instead of dealing with ratio as $(1/n_i^2)$ it is actually convenient to define the inverse of the permittivity, which is called the impermeability $\hat{\eta}$ such that

$$\hat{\eta} \cdot \hat{\epsilon}_r = \mathbf{1} \Rightarrow \mathbf{E} = \frac{1}{\epsilon_0} \hat{\eta} \mathbf{D} \quad (3.4)$$

Using the impermeability tensor, the index ellipsoid is then written as

$$\eta_{ij}(E) x_i^2 = 1 \quad (3.5)$$

In this form, applying an external electric field will lead to a small variation of the refractive index, which can easily be treated as a perturbation of the impermeability tensor $\hat{\eta}$. Interestingly, such effect can be advantageously used to control either the intensity or the phase of the propagating radiation. Two cases are important:

- *Pockels effect*: the variation of the refractive index is linear with the applied electric field. It can only occur with *non-centro symmetric* crystal.
- *Kerr effect*: the variation of the refractive index is quadratic with the applied electric field.

The perturbation that are created yield a modification of the impermeability tensor

$$\eta_{ij}(E) = \eta_{ij}(0) + \sum_k \left(\frac{\partial \eta_{ij}}{\partial E_k} \right)_{E=0} E_k + \frac{1}{2} \sum_{k,\ell} \left(\frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_\ell} \right) E_k E_\ell + \dots \quad (3.6)$$

It is common to redefine

$$r_{ijk} = \left(\frac{\partial \eta_{ij}}{\partial E_k} \right)_{E=0} \quad \text{and} \quad s_{ijkl} = \frac{1}{2} \left(\frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_\ell} \right) E_k E_\ell \quad (3.7)$$

such that

$$\eta_{ij}(E) = \eta_{ij}(0) + \sum_{k=x,y,z} r_{ijk} E_k + \sum_{k,\ell=x,y,z} s_{ijkl} E_k E_\ell \quad (3.8)$$

Finally, in order to avoid the sum in the equation, it is important to use the Einstein notation

$$\begin{aligned} \sum_{k=x,y,z} r_{ijk} E_k &\Rightarrow r_{ijk} E_k \\ \sum_{k,\ell=x,y,z} s_{ijkl} E_k E_\ell &\Rightarrow s_{ijkl} E_k E_\ell \end{aligned}$$

3.1.3 A bit of symmetry

Since $\hat{\epsilon}_r$ is a symmetric tensor, then $\hat{\eta}$ is also a symmetric tensor: $\eta_{ij} = \eta_{ji}$:

$$\hat{\epsilon}_r = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{bmatrix} = \begin{bmatrix} \epsilon_1 & \epsilon_6 & \epsilon_5 \\ \epsilon_6 & \epsilon_2 & \epsilon_4 \\ \epsilon_5 & \epsilon_4 & \epsilon_3 \end{bmatrix} \quad (3.10)$$

This 3×3 tensor can then be expressed as a vector of dimension 6. This is called the Voigt notation. Of course we can use this notation for either the susceptibility tensor or the impermeability tensor.

$$\hat{\epsilon}_r = \begin{pmatrix} \epsilon_1 = \epsilon_{11} \\ \epsilon_2 = \epsilon_{22} \\ \epsilon_3 = \epsilon_{33} \\ \epsilon_4 = \epsilon_{23} = \epsilon_{32} \\ \epsilon_5 = \epsilon_{13} = \epsilon_{31} \\ \epsilon_6 = \epsilon_{12} = \epsilon_{21} \end{pmatrix}; \quad \eta = \begin{pmatrix} \eta_1 = \eta_{11} \\ \eta_2 = \eta_{22} \\ \eta_3 = \eta_{33} \\ \eta_4 = \eta_{23} = \eta_{32} \\ \eta_5 = \eta_{13} = \eta_{31} \\ \eta_6 = \eta_{12} = \eta_{21} \end{pmatrix} \quad (3.11)$$

Tensor for Pockels effect

Similarly we can use the Voigt notation to reduce the [$3 \times 3 \times 3 = 27$ elements] tensor relative to the Pockels effect r_{ijk} such that the impermeability tensor becomes:

$$\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix} (E) = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix}_{(0)} + \begin{pmatrix} \Delta \left(\frac{1}{n^2} \right)_1 \\ \vdots \\ \vdots \\ \Delta \left(\frac{1}{n^2} \right)_6 \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix}_{(0)} + \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & \\ r_{31} & & \\ r_{41} & & \\ r_{51} & & \\ r_{61} & & \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \quad (3.12)$$

which is the very same equation as eq. (3.6).

Tensor for Kerr effect

As for the tensor relative to the Pockels effect, the tensor for the Kerr effects has symmetries than can be exploited in order to considerably reduced its dimension. Note that without taking into account those symmetries, the total dimension of that tensor is $3 \times 3 \times 3 \times 3 = 81$ elements!

First of all $s_{ijkl} = s_{jikl}$, but also since

$$\left(\frac{\partial \eta_{ij}}{\partial E_k \partial E_\ell} \right) \equiv \left(\frac{\partial \eta_{ij}}{\partial E_\ell \partial E_k} \right)$$

then $s_{ijkl} = s_{ijlk}$. Therefore the original tensor can be expressed by a 6×6 tensor:

$$\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix} (E) = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix}_{(0)} + \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{21} & s_{22} & \dots & & & \\ s_{31} & \vdots & \ddots & & & \\ s_{41} & & & & & \\ s_{51} & & & & & \\ s_{61} & & & & & s_{66} \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \\ 2E_y E_z \\ 2E_x E_z \\ 2E_x E_y \end{pmatrix} \quad (3.13)$$

Hopefully, we will be able to reduce these tensors even more...

3.2 The linear electro-optics effect

The linear electro-optics effect can only appear with non centro-symmetric crystal, which do not posses the inversion symmetry. We can use the very same argument that we use to prove that only non centro-symmetric crystal can exhibit a second order nonlinearity $\chi^{(2)}$. Let assume that we use a centro-symmetric crystal and apply a field such that it yields a change of refractive index $\Delta n_1 = sE$ where s is a proportionality constant characteristic of the electro-optic effect. If we reverse the electric field the change in refractive index will be $\Delta n_2 = s(-E)$. Since the crystal posses the inversion symmetry property, we must have $\Delta n_1 = \Delta n_2$ which occurs if and only if $s = 0$. Therefore centro-symmetric crystal can only exhibit quadratic electro-optic effect. Non centro-symmetric crystal on the other hand can also present linear electro-optic effect.

3.2.1 Example: KDP

KDP (KH_2PO_4) is a uniaxial crystal. As for other uniaxial crystal the susceptibility tensor (and therefore the impermeability) tensor presents two distinct eigenvalues:

$$\hat{\eta} = \begin{pmatrix} \frac{1}{n_o^2} & 0 & 0 \\ 0 & \frac{1}{n_o^2} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{pmatrix} \quad (3.14)$$

And the index ellipsoid is described by the equation:

$$\frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \quad (3.15)$$

In the presence of an external electric field there is a modification of the refractive indices according to

$$\begin{pmatrix} \Delta \left(\frac{1}{n^2} \right)_1 \\ \vdots \\ \vdots \\ \Delta \left(\frac{1}{n^2} \right)_6 \end{pmatrix} = r_{ij} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \quad (3.16)$$

where in the particular case of KDP, due to the symmetry properties of the crystal yield the electro-optic tensor has the following form

$$r_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \quad (3.17)$$

Electric field along the z -direction

Considering that the applied field is $\mathbf{E} = E_z \mathbf{e}_z$, we can readily compute the impermeability tensor η by adding eq. (3.14) and eq. (3.16) to get

$$\hat{\eta}(E) = \begin{pmatrix} \frac{1}{n_o^2} & r_{63} E_z & 0 \\ r_{63} E_z & \frac{1}{n_o^2} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{pmatrix} \quad (3.18)$$

From this impermeability tensor we can calculate its eigenvalues and eigenvectors. As previously the eigenvectors will represents the principal axis of the crystal and the eigenvalues the refractive indices along these axis.

Eigenvalues The eigenvalues are calculated by solving the equation $\det(\hat{\eta} - \lambda \mathbf{1}) = 0$. This is simply given by

$$\begin{aligned} \left(\frac{1}{n_e^2} - \lambda\right) \left[\left(\frac{1}{n_o^2} - \lambda\right)^2 - (r_{63} E_z)^2 \right] = \\ \left(\frac{1}{n_e^2} - \lambda\right) \left(\frac{1}{n_o^2} + r_{63} E_z - \lambda\right) \left(\frac{1}{n_o^2} - r_{63} E_z - \lambda\right) = 0 \end{aligned} \quad (3.19)$$

It is clear that the eigenvalues of the impermeability tensor are

$$\begin{cases} \lambda_1 = 1/n_o^2 + r_{63} E_z \\ \lambda_2 = 1/n_o^2 - r_{63} E_z \\ \lambda_3 = 1/n_e^2 \end{cases} \quad (3.20)$$

The change of refractive index can be calculated by

$$\frac{1}{n_u^2} = \frac{1}{n_o^2} + d\left(\frac{1}{n_o^2}\right) = \frac{1}{n_o^2} + r_{63} E_z \quad (3.21)$$

If we assume that $r_{63} E_z \ll 1/n_o^2$ and calculating the differentiation $d(1/n^2) = (-2/n^3) dn$ we readily find the change of refractive index:

$$dn = \frac{-n_o^3}{2} r_{63} E_z \quad (3.22)$$

and therefore the new refractive indices (along \mathbf{u} and \mathbf{v}) are

$$n_{\mathbf{u}} = n_o - \frac{1}{2} n_o^3 r_{63} E_z \quad (3.23)$$

$$n_{\mathbf{v}} = n_o + \frac{1}{2} n_o^3 r_{63} E_z \quad (3.24)$$

Eigenvectors The eigenvector corresponding to the eigenvalues are

$$\mathbf{u} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}; \quad \mathbf{v} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}; \quad \mathbf{w} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (3.25)$$

the z-axis is not changed but the x – (resp. y –) direction are rotated by 45° . Finally we have the new ellipsoid of indices which can be mathematically described by

$$\left(\frac{1}{n_o^2} + r_{63} E_z\right) u^2 + \left(\frac{1}{n_o^2} - r_{63} E_z\right) v^2 + \frac{1}{n_e^2} z^2 = 1 \quad (3.26)$$

is plotted on Fig. 3.1. Note that we could write this as

$$\frac{u^2}{n_u^2} + \frac{v^2}{n_v^2} + \frac{z^2}{n_e^2} = 1$$

In the presence of an external electric field, the uniaxial crystal of KDP is now a biaxial crystal!

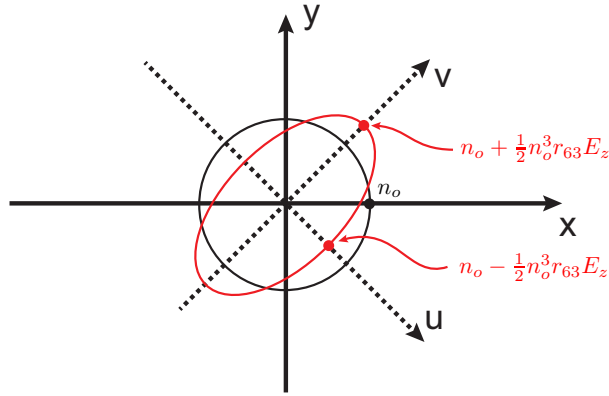


Figure 3.1: Index ellipsoid for KDP without any external field (black) and with an external field applied along the z -direction.

3.2.2 Example: Li-Niobate

The lithium niobate (LiNbO_3) is another important uniaxial crystal used for its electro-optical properties. It belongs to a different group point symmetry than the KDP crystal (C_{3v}). Due to this symmetry group the linear electro-optic tensor has the following form:

$$r_{ij} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \quad (3.27)$$

As previously let us assume that the external field is along the optic axis of the crystal. The equation of the index ellipsoid are in that case:

$$\left(\frac{1}{n_o^2 + r_{13}E} \right) x^2 + \left(\frac{1}{n_o^2 + r_{13}E} \right) y^2 + \left(\frac{1}{n_e^2 + r_{33}E} \right) z^2 = 1 \quad (3.28)$$

By contrast with the case of KDP, the applied field does not lead to a change of orientation of the principal axis of the crystal! However the length of the semiaxis of the ellipsoid are modified according to:

$$\begin{cases} n_x = n_o - \frac{1}{2} n_o^3 r_{13} E \\ n_y = n_o - \frac{1}{2} n_o^3 r_{13} E \\ n_z = n_e - \frac{1}{2} n_e^3 r_{33} E \end{cases} \quad (3.29)$$

Let us now consider that the external field is applied along the x -axis of the crystal. The induced birefringence is then given by

$$n_z - n_y = (n_e - n_o) - \frac{1}{2} (n_e^3 r_{33} - n_o^3 r_{13}) E \quad (3.30)$$

3.2.3 General case

In general an external electric field causes a modification of the refractive indices and even the orientation of the principal axis of the crystal. This appears as mixed term in the

equation of the ellipsoid of indices. In its most general formulation the equation of the ellipsoid of indices is

$$\left(\frac{1}{n_x^2} + r_{1k}E_k\right)x^2 + \left(\frac{1}{n_y^2} + r_{2k}E_k\right)y^2 + \left(\frac{1}{n_z^2} + r_{3k}E_k\right)z^2 + 2yz r_{4k}E_k + 2xz r_{5k}E_k + 2xy r_{6k}E_k = 0 \quad (3.31)$$

where the summation convention has to be used. Namely k represents the three coordinates x, y, z so that $k = 1, 2, 3$ and $a_k b_k = \sum_{k=1}^3 a_k b_k$.

3.2.4 Applications of the linear electro-optic effect

As we saw previously the application of an external electric field can change significantly the optical properties of the crystal. These effects can be efficiently used to modify and control the propagating light, in particular by affecting its polarisation state. It is also possible to use those crystals to encode information on the travelling field either by phase or amplitude modulation. We are looking here at a few possible applications of the linear electro-optic effect.

The Pockels cell

The way to use an electro-optic crystal is illustrated in its more basic way on Fig. 3.2. The main idea is to send a polarised beam onto the crystal. As we saw previously the application of an external static field yields a change of the refractive indices and possibly even modify the nature of the crystal¹.

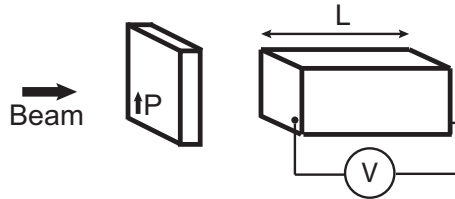


Figure 3.2: Principle of a Pockel cell. A polariser indicated by the plate P is placed before the electro-optic crystal.

Induced birefringence and *retardation*

Let us consider a KDP crystal. The application of an external field along the z -axis of a KDP crystal modifies the orientation of its principal axis. In particular in the plane $z = 0$ the axes are rotated by $\pi/4$ (see Section. 3.2.1 and Fig. 3.1). Let us assume that we use a beam polarised along the x -axis of the crystal. In the new referential (\mathbf{u}, \mathbf{v}) the field decomposes along both axes:

$$E_u = A \exp[i(k_u z - \omega t)] \quad (3.32a)$$

$$E_v = A \exp[i(k_v z - \omega t)] \quad (3.32b)$$

$$(3.32c)$$

¹An uniaxial KDP crystal can turn into a biaxial crystal under the presence of an external field.

and at the end of the crystal, which has a length ℓ :

$$E_u(\ell) = A \exp \left\{ i \left[\frac{\omega}{c} \left(n_o - \frac{1}{2} r_{63} n_o^3 E_z \right) \ell - \omega t \right] \right\} \quad (3.33a)$$

$$E_v(\ell) = A \exp \left\{ i \left[\frac{\omega}{c} \left(n_o + \frac{1}{2} r_{63} n_o^3 E_z \right) \ell - \omega t \right] \right\} \quad (3.33b)$$

As we see from eq. (3.33), as the field propagates both component acquire a different phase leading to a net phase-difference

$$\Delta\phi = \phi_u - \phi_v = k \cdot \ell \cdot \Delta n = \frac{\omega}{c} n_o^3 r_{63} E_z \ell = \frac{\omega}{c} n_o^3 r_{63} V \quad (3.34)$$

where the difference of potential $V = E \cdot \ell$. This net phase difference is called the *retardation*.

half-wave voltage

For a KDP crystal the retardation induced by the external electric field applied along the z -axis is given at eq. (3.34)

$$\Delta\phi = \frac{\omega}{c} n_o^3 r_{63} E_z \ell = \frac{2\pi}{\lambda} n_o^3 r_{63} V$$

which obviously depends not only on the length of the crystal ℓ but also on the strength of the applied difference of potential V . Using a field polarised along the x -axis (Fig. 3.1) of the crystal the field is described by the eq. (3.32), which we rewrite in a simpler form as

$$E_u = A \cos \omega t \quad (3.35a)$$

$$E_v = A \cos (\omega t - \Delta\phi) \quad (3.35b)$$

Depending on the value of the retardation $\Delta\phi$ the output beam can have various states of polarisation. As shown on Fig. 3.3 there is a particular case for $\Delta\phi = \pi$. For this case

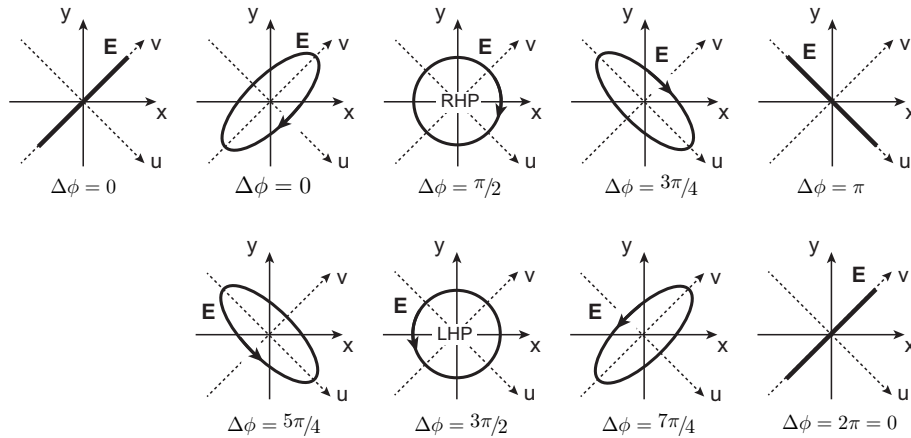


Figure 3.3: States of polarisation at the output of the electro-optical crystal according to the applied retardation $\Delta\phi$.

the field is linearly polarised (as initially) but is not along the \mathbf{u} -axis.

In the particular case of $\Delta\phi = \pi$ the crystal acts as a half-wave plate². Since the induced phase is equal to π we call this particular voltage the *half-wave voltage* V_π :

$$V_\pi = \frac{\lambda}{2n_o^3 r_{63}} \quad (3.36)$$

In the particular case of KDP $r_{63} = 10.6 \times 10^{-12} \text{ m/V}$ and $n_o = 1.47$ at $\lambda = 632 \text{ nm}$. The corresponding half-wave voltage is then (eq. (3.36)) $V_\pi = 9.3 \text{ kV}$. This is actually really not a small tension! The speed for modulating such a tension is limited to a few kHz. Using the expression for the half-wave voltage we can express the retardation in a general form

$$\Delta\phi = \pi \frac{V}{V_\pi} \quad (3.37)$$

Use of the linear electro-optic effect for amplitude modulation

As shown by the eq. (3.33) the application of an external electric field yields a birefringence, which causes the polarisation to evolve along the crystal. We previously saw that in the case of beam polarised linearly along the x -axis, the application of a voltage equal to V_π would lead to a beam linearly polarised along the y -axis. Suppose now that we place a polarise after the crystal. The axis of the polarise is along the y -axis. In the presence of the external electric field, the polarisation of the input beam has rotated and is now aligned with the axis of the polarise: the beam goes through without any loss. On the other hand when the external electric field is off the linear polarisation of the input beam remains along the x -axis and is 90° rotated with respect to the polarise: the beam is blocked.

In a more general way we can use a polarise in combination with an electro-optical crystal in order to modulated the amplitude of a beam. A typical arrangement for such scheme is presented on Fig. 3.4. For this scheme the input polarisation is set along the x -axis of the crystal.

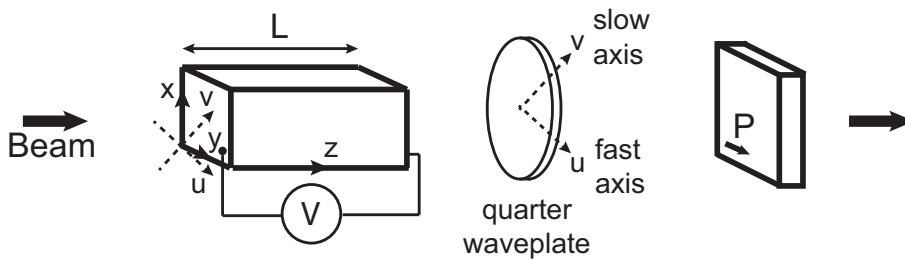


Figure 3.4: Typical scheme for amplitude modulation

Since the input polarisation is at 45° with respect to the principal axis of the crystal (in the presence of an external electric field) the projection of the input field along the principal axis of the crystal are

$$E_u = A \cos \omega t \quad (3.38a)$$

$$E_v = A \cos \omega t \quad (3.38b)$$

²We remind that a half-wave plate placed at an angle α with respect to the optics axis rotates a linear polarisation by an angle equal to 2α .

Use of the linear electro-optic effect for phase modulation

Let us consider now that the input beam is polarised along the u -axis of the electro-optical crystal. In this case the component along the v -axis of the crystal is null. The application of the external electric field does not modify the polarisation state of the beam but induces a change of the phase following

$$\Delta\phi_u = \frac{-\omega}{c} \Delta n_u \ell = \frac{\omega n_o^3 r_{63}}{2c} E_z \ell \quad (3.45)$$

If the bias field is sinusoidal ($E_z = E_m \sin \omega_m t$), then an incident field assumed to be a plane wave (i.e. $E_{in} = A \cos \omega t$), will propagate through the crystal acquiring a sinusoidal phase. At the output plane the electric field associated with the light beam will be

$$E_{out} = A \cos \left[\omega t - \frac{\omega}{c} \left(n_o - \frac{n_o^3}{2} r_{63} E_m \sin \omega_m t \right) \ell \right] \quad (3.46)$$

And if we drop the constant factor we have the expression of the output field:

$$E_{out} = A \cos (\omega t + \delta \sin \omega_m t) \quad (3.47)$$

where δ is referred in literature as the phase modulation index and its expression is given by the following relation:

$$\delta = \frac{\omega n_o^3 r_{63} E_m \ell}{2\lambda} = \frac{\pi n_o^3 r_{63} E_m \ell}{\lambda} \quad (3.48)$$

The light beam then experiences phase modulation because of its propagation inside the crystal in such a configuration. The depth of the phase modulation is represented by the phase modulation index δ . A scheme of an electro-optic phase modulator is shown in Fig. 3.6.

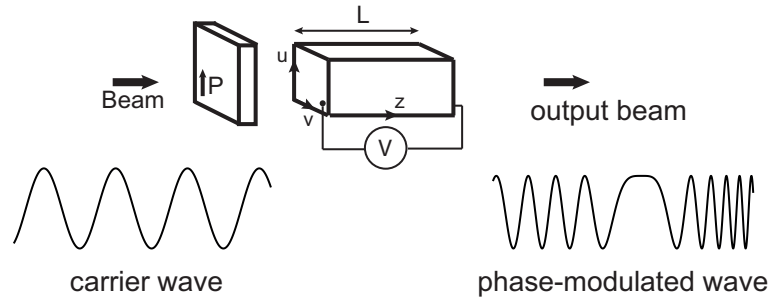


Figure 3.6: typical setup for phase-modulation of a optical beam

Appendix A

Electro-optic tensor depending of symmetry

There exists 32 different classes of crystal. For each class the susceptibility tensor has different vanishing coefficients [Boyd].

A.0.1 Triclinic symmetry

Class 1

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix}$$

A.0.2 Monoclinic symmetry

class 2

$$\begin{matrix} axis \parallel \mathbf{y} & \begin{bmatrix} 0 & r_{12} & 0 \\ 0 & r_{22} & 0 \\ 0 & r_{32} & 0 \\ r_{41} & 0 & r_{43} \\ 0 & r_{52} & 0 \\ r_{61} & 0 & r_{63} \end{bmatrix} & axis \parallel \mathbf{z} & \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ r_{51} & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \end{matrix}$$

class m

$$\begin{matrix} m \perp \mathbf{y} & \begin{bmatrix} r_{11} & 0 & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & r_{53} \\ 0 & r_{62} & 0 \end{bmatrix} & m \perp \mathbf{z} & \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ r_{31} & r_{32} & 0 \\ 0 & 0 & r_{43} \\ 0 & 0 & r_{53} \\ r_{61} & r_{62} & 0 \end{bmatrix} \end{matrix}$$

A.0.3 Orthorhombic symmetry

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$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

2mm

$$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A.0.4 Tetragonal symmetry

class 4 and $\bar{4}$

$$\begin{array}{cc} \text{class 4} & \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix} & \text{class } \bar{4} & \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & -r_{13} \\ 0 & 0 & 0 \\ r_{41} & -r_{51} & 0 \\ r_{51} & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \end{array}$$

class 422 and 4mm

$$\begin{array}{cc} \text{class 422} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix} & \text{class 4mm} & \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

class $\bar{4}2m$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

crystal	r_{ij} $\times 10^{-22} \text{m.V}^{-1}$
KDP	$r_{41} = 8.77$ $r_{63} = 10.3$
KD*P	$r_{41} = 8.8$ $r_{63} = 26.8$
ADP	$r_{41} = 23.76$ $r_{63} = 8.56$
AD*P	$r_{41} = 40$ $r_{63} = 10$

A.0.5 Cubic symmetry

class $\bar{4}3\text{m}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

class 432 \implies actually all coefficients vanish for crystal with this symmetry.

A.0.6 Trigonal symmetry

class 3 and 32

$$\text{class 3} \quad \begin{bmatrix} r_{11} & -r_{22} & r_{13} \\ -r_{11} & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ -r_{22} & -r_{11} & 0 \end{bmatrix} \quad \text{class 32} \quad \begin{bmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{bmatrix}$$

class 3m

$$m \perp \mathbf{x} \quad \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix} \quad m \perp \mathbf{y} \quad \begin{bmatrix} r_{11} & 0 & r_{13} \\ -r_{11} & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & -r_{11} & 0 \end{bmatrix}$$

crystal	r_{ij} $\times 10^{-22} \text{m.V}^{-1}$
LiNbO ₃	$r_{13} = 9.6$
	$r_{22} = 6.8$
	$r_{33} = 30.9$
	$r_{51} = 31.6$

A.0.7 Hexagonal symmetry

class 6, 6mm, 622 and $\bar{6}$

$$\text{class 6} \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{class 6mm} \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{class 622} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{class } \bar{6} \begin{bmatrix} r_{11} & -r_{22} & 0 \\ -r_{11} & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & -r_{11} & 0 \end{bmatrix}$$

class $\bar{6}m2$

$$m \perp \mathbf{x} \begin{bmatrix} 0 & -r_{22} & 0 \\ 0 & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix} \quad m \perp \mathbf{y} \begin{bmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r_{11} & 0 \end{bmatrix}$$