
Modern Optics II: Nonlinear Optics

SHEET III

$\chi^{(2)}$ -processes

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Exercise 1 *Invariant in Second Harmonic Generation*

For perfect phase-matching, the coupled equation for the second harmonic generation :

$$\frac{d\rho_1}{d\zeta} = \rho_1\rho_2 \sin \theta \quad (1a)$$

$$\frac{d\rho_2}{d\zeta} = -\rho_1^2 \sin \theta \quad (1b)$$

$$\frac{d\theta}{d\zeta} = \left(2\rho_2 - \frac{\rho_1^2}{\rho_2}\right) \cos \theta + \ell\Delta k \quad (1c)$$

lead to the invariant :

$$\rho_1^2\rho_2 \cos \theta = G = \text{cste} \quad (2)$$

To demonstrate this, we started from the eq. (1c), written as

$$\frac{d\theta}{d\zeta} = \frac{\cos \theta}{\sin \theta} \frac{d}{d\zeta} [\ln(\rho_1^2\rho_2)] + \ell\Delta k \quad (3)$$

1. Demonstrate the equation (3).
2. Find the invariant in the case of imperfect phase-matching $\Delta k \neq 0$

Exercise 2

We want to realise the second harmonic of a 1064 nm laser using a KDP nonlinear crystal. The nonlinear coefficient is $d_{\text{eff}} = 4.35 \times 10^{-13}$ m/W. The refractive indices at the fundamental and the second harmonic are respectively $n(1064) = 1.4938$ and $n(532) = 1.5123$. We focus a 100 fs pulse laser (1 nJ) energy into the 3 mm long crystal. We assume that the beam is 30 μm diameter at the focus point at the fundamental ($\lambda = 1064$ nm). We suppose that we have perfect phase-matching. Calculate the intensity of the second harmonic beam at the output of the crystal. Is the assumption of weak depletion of the pump valid?

Exercise 3

Let's consider a crystal, that presents a 3^{rd} -order electric susceptibility such that the induced atomic polarisation is $P_{NL} = \epsilon_0\chi^{(3)}E^3$. For an input field

$$\mathbf{E} = F_1e^{i\omega_1t} + F_2e^{i\omega_2t} + F_3e^{i\omega_3t} + c.c.$$

- From symmetry considerations, find the amplitudes of the contributions at $\omega_1, \omega_2, \omega_3, 3\omega_1, 3\omega_2, 3\omega_3, (\omega_1 + \omega_2 + \omega_3), (\omega_1 + \omega_2 - \omega_3), (\omega_1 - \omega_2 + \omega_3), (-\omega_1 + \omega_2 + \omega_3), (2\omega_1 \pm \omega_2), (2\omega_1 \pm \omega_3)$.
note : you may start with a χ^2 crystal, and $E = E_1e^{i\omega_1t} + E_2e^{i\omega_2t} + c.c.$
- Check that $p(2\omega_i)$ and $P(0)$ are null.