Modern Optics II: Nonlinear Optics
Perturbation theory

Exercises' sheet No 1 May 2017

<u>**Exercise**</u> **1** *Lorentz model with symmetric potential*

In a symmetric potential, the Lorentz model can be written :

$$\ddot{q} + \epsilon\gamma\dot{q} + \omega_0^2 q(1 + \epsilon q^2) = A\exp(j\omega t) + c.c. \tag{1}$$

1. seek for a solution as $q(t) = q_0(t) + \epsilon q_1(t)$
2. what is/are the new frequency that can appear ?
3. derive the expression for $\chi^{(3)}[\omega]$ and $\chi^{(3)}[3\omega]$

<u>**Exercise**</u> **2** *Resonance in symmetric potential*

1. Consider the following damped oscillator :

$$\ddot{q} + \gamma\dot{q} + \omega_0^2 q = 0 \tag{1}$$

   Use the change of variable $\theta = \omega_0 t$ in eq. (**??**). What is the only parameter that characterize this oscillator ? How does this parameter characterize the resonance of the system ?
   <u>*hint*</u> *: Use a driving force A $e^{j\omega t)}$ to study the resonance of the system.*

2. *Let's now consider the following oscillator*

$$\ddot{q} + \omega_0^2 q(1 + \epsilon q_2) = 0 \tag{2}$$

   (a) *Show that the traditional perturbative approach ($q(t) = q_0(t) + \epsilon q_1(t)$) fails.*

   (b) *Such problem was solved by Poincaré and Lindstedt, who introduce the following rescalling of the time :*

$$t = s(1 + \epsilon\omega_1 + \epsilon^2\omega_2 + \dots) \tag{3}$$

   *Use this approach (at the first order) to solve the problem (eq. (**??**). What should be the condition for ω_1 ? How does this influence the resonance of the oscillator ?*

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