Frequency analysis of optical imaging system.

- 1st scientist to use Fourier approach in optics: P.M. Duffieux (1946)
- In the US Otto Schade (electrical engineer) employed the method of linear Σ in analysis and improvement of television camera lenses
- In UK H.H. Hopkins: first calculation of transfer function in the presence of aberrations

Actually the Fourier analysis was used much earlier by Ernst Abbe (Jena) (1840-1905) and Lord Rayleigh

$\Rightarrow$ here: role of Fourier analysis in the theory of coherent & incoherent imaging.
i.e. holography, microscopy ..

I] Generalized treatment of imaging systems.

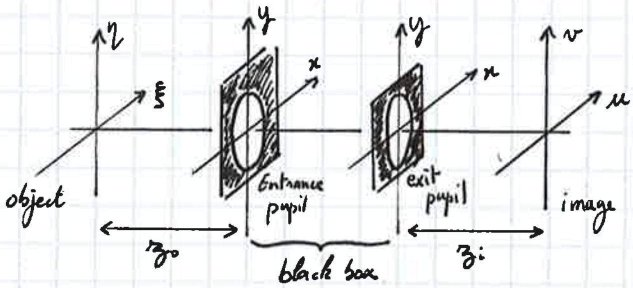
We saw last week the case of a single lens, under monochromatic illumination ($f>0$; convergent lens)
now $\Rightarrow$ Σ more complicated
not necessary monochromatic.

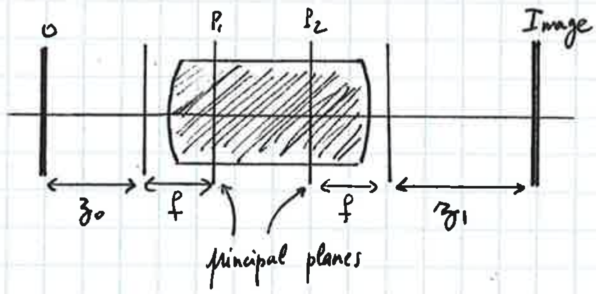
a) generalized model

- Suppose that we have a more complicated system (many lenses, not necessarily positive).

Assumption $\Rightarrow$ the system produces a real image (note that if it produces a virtual image, the lens of your eye will then produce a real image on the retina)

it is now a "black box". We only know the entrance and exit pupils.





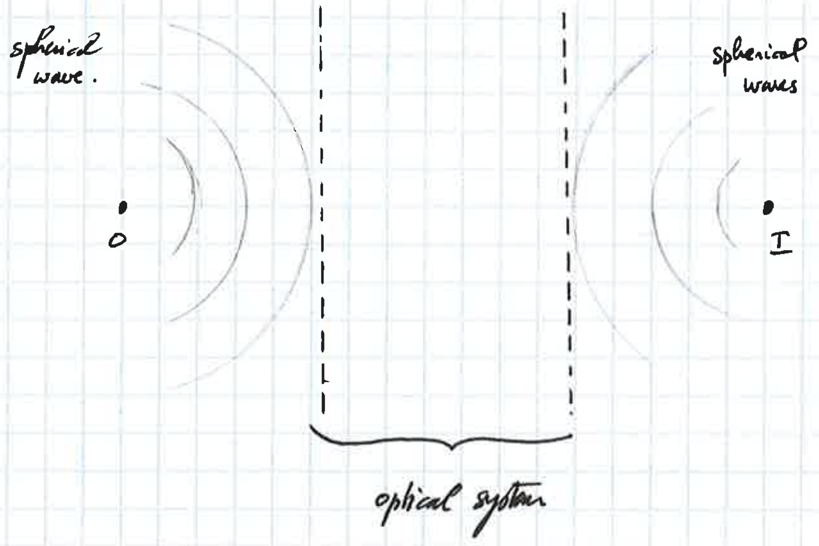
we assume that our system can be described by geometrical optics $\Rightarrow$ we can visualize it as a system with 2 principal planes)

Entrance and exit pupil are in fact images of the same limiting aperture within the optical system. This is a way to introduce spatial limitations of the wavefront that ultimately leads to diffraction

z_i: distance from the 2nd principal plane to the image is the distance taken into account in diffraction eq. to represent the effect of diffraction by the exit pupil on the point-spread function of the optical system.

Imaging Σ is "diffraction-limited" if a diverging spherical wave, emanating from a point-source object, is converted by the system into a new wave, again perfectly spherical, that converges toward an ideal point in the image plane. Magnification (factor in size) must be the same for all points in the image field if the system is ideal.

ideal diffraction limited imaging system:



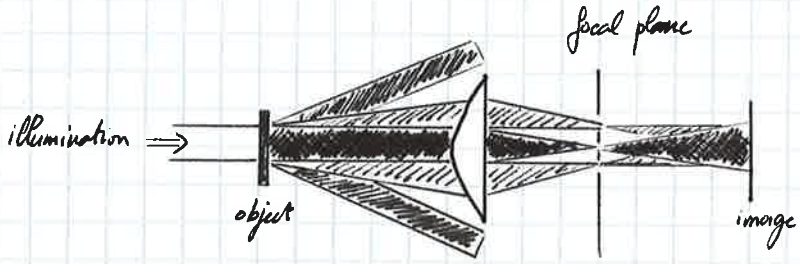
For any real imaging system, this properties will be satisfied, at best, over only finite regions of the object plane and the image plane. Within these regions, the Σ can be regarded as diffraction-limited.

Effects of diffraction on the image.

The main assumption is that geometrical optics describes perfectly the propagation of light inside the optical Σ i.e. between the entrance and exit pupils. Therefore diffraction effects only play a role during the passage of light from the object to the entrance pupil and from the exit pupil to the image

Since the 2 pupils are image of each other, regarding the image resolution as being limited by the entrance pupil is equivalent as being limited by the exit pupil!

The 1st one to look at the effects of diffraction resulting from the entrance pupil was Ernst Abbe (1873) in the context of coherent imaging with microscopy –



the object is viewed as a diffraction gratting

↳ high-frequency components may not be intercepted by the optical system (lens)

As we say, we can also look at the problem from the exit pupil (Lord Rayleigh's approach). For a monochromatic illumination the image amplitude is given by a superposition integral

$$U_i(u,v) = \iint_{-\infty}^{+\infty} \underbrace{h(u,v;\xi,\eta)}_{} \; \underbrace{U_o(\xi,\eta)}_{} \, d\xi \, d\eta$$

amplitude at the image coordinates (u,v) in response to a pt-source object at (ξ,η)

amplitude distribution transmitted by the object.

If there is no aberrations, the response h arises from a spherical wave converging from the exit pupil towards the image point ($u = M\xi$, $v = M\eta$) with M the magnification factor. ($M<0$ if image is reversed)

The light amplitude of an ideal point is simply the Fraunhofer diffraction pattern of the exit pupil centred on image coordinate ($u = M\xi$, $v = M\eta$)

coord. in the plane of the exit pupil

$$\Rightarrow h(u,v;\xi,\eta) = \frac{A}{\lambda z_i} \iint_{-\infty}^{+\infty} P(x,y) \exp\left\{-\frac{i2\pi}{\lambda z_i}\left[(u-M\xi)x + (v-M\eta)y\right]\right\} dx\,dy$$

amplitude point-spread function

distance from exit pupil to image plane

pupil function as usual

$$\begin{cases} P(x,y) = 1 & \text{inside the aperture} \\ P(x,y) = 0 & \text{otherwise} \end{cases}$$

Note: we didn't take quadratic phase factor into account here.

To achieve space invariance in the imaging operation we need to remove the effects of magnification in the eq.

$$\Rightarrow \tilde{\xi} = M.\xi \quad \text{and} \quad \tilde{\eta} = M\eta$$

then the amplitude PSF becomes:

$$h(u-\tilde{\xi}, v-\tilde{\eta}) = \frac{A}{\lambda z_i} \iint_{-\infty}^{+\infty} P(x,y) \exp\left\{-\frac{i2\pi}{\lambda z_i}\left[(u-\tilde{\xi})x + (v-\tilde{\eta})y\right]\right\} dx\,dy$$

And the ideal image predicted by geometrical optics is defined as:

$$U_{geom}(\tilde{\xi},\tilde{\eta}) = \frac{1}{|M|}\, U_o\left(\frac{\tilde{\xi}}{M}, \frac{\tilde{\eta}}{M}\right)$$

(simply scaling by M!)

therefore

$$U_i(u,v) = \iint_{-\infty}^{+\infty} h(u-\tilde{\xi}, v-\tilde{\eta})\, U_{geom}(\tilde{\xi},\tilde{\eta})\, d\tilde{\xi}\, d\tilde{\eta}$$

where

$$h(u,v) = \frac{A}{\lambda z_i} \iint_{-\infty}^{+\infty} P(x,y) \exp\left[-\frac{i2\pi}{\lambda z_i}(ux + vy)\right] dx\,dy$$

In general for a diffraction-limited Σ, we can regard the image as being a convolution of the image predicted by geometrical optics with an impulse response ( ) that is the Fraunhofer diffraction pattern of the exit pupil (PSF)