Non linear phase-shift (Self-phase modulation)

Let us now assume that we do not have any dispersive effects.

$$i\partial_z A = \quad + \gamma |A|^2 A \qquad (1)$$

$$A = \sqrt{P_0}\, U \Rightarrow \quad i\partial_z U = \gamma P_0 |U|^2 U \quad (2) \Rightarrow \text{Non linear length } \boxed{L_{NL} = \frac{1}{\gamma P_0}}$$

$\sqrt{P_0}$: peak power

if we know introduce $U = Ve^{i\phi_{NL}}$ and insert in (2) we obtain:

$$i\frac{\partial V}{\partial z} e^{i\phi_{NL}} - \frac{\partial \phi_{NL}}{\partial z} \cdot V e^{i\phi_{NL}} = \frac{V^2}{L_{NL}} \cdot V e^{i\phi_{NL}}$$

$\Rightarrow$ Real part:

$$-\frac{\partial \phi_{NL}}{\partial z} = \frac{V^2}{L_{NL}}$$

Imaginary part:

$$\frac{\partial V}{\partial z} = 0 \quad \leftarrow \text{ the amplitude doesn't change during the propagation!}$$

therefore the eq. for the phase is readily solved:

$$\phi_{NL}(z) = -\frac{V^2}{L_{NL}} \cdot z =$$

finally the solution of (2) is $\boxed{U(z,T) = U(0,T)\exp[+i\phi_{NL}(z,T)]}$

and $\phi_{NL} = -|U(0,T)|^2 \frac{z}{L_{NL}}$ intensity-dependent phase-shift but the pulse shape (in time) doesnot change! (SPM)

As previously (dispersive case) we can look at the effect of SPM on the chirp:

$$\delta\omega = \frac{\partial\phi}{\partial T} = -\left(\frac{\gamma}{L_{NL}}\right)\frac{\partial}{\partial T}|U(0,T)|^2$$

of course $\delta\omega$ depends on the initial chirp of $(U(0,T))$.

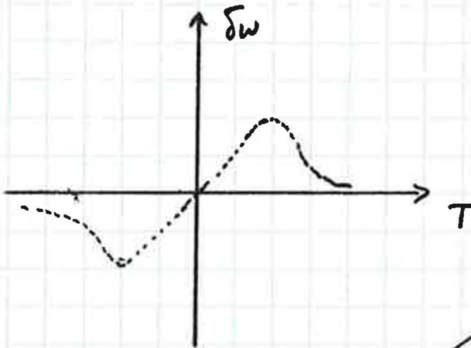
for no initial chirp:

Note that the frequency chirping ($\delta\omega$) induced by SPM increases with the propagation distance.

⇒ New-frequency component are continuously generated as the pulse propagates down the fiber!

↳ the initial spectrum broadens as it propagates!

Ex: Gaussian pulse. $U(0,T) = \exp\left(-\frac{T^2}{2T_0}\right) \rightarrow \delta\omega = \left(\frac{T}{T_0}\right)\cdot\frac{\gamma}{L_{NL}}\exp\left(-\frac{T^2}{2T_0^2}\right)$



Note that the pulse is almost linear around the center of the pulse ($T=0$) and positive. We can see that for anomalous dispersion regime, the SPM induced (chirp may be balanced by the dispersive induced chirp. In normal dispersion → not possible (same variation)

(solitonic effect)

Interplay between dispersive and N.L. effects.

(1) Modulational instability.

Many systems exhibit an instability that can lead to the modulation of the steady state. As we will see pumping with CW wave an optical fiber an actually leads to the appearance of sidebands in the spectrum $\rightarrow$ modulation in the time domain.

Actually such appearance of modulation was studied in many branches of physics : ~~hydrodynamics~~, plasma physics, fluid dynamics, NLO.

Linear stability analysis.

The idea here is to look at how a perturbation (fluctuation of the amplitude) can yield the appearance of sidebands. We start with the NLSE.

$$i\partial_z A = -\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A \quad (1)$$

We consider here that A is CW [Therefore A is independent of T at the input : $A(z=0,T) = A(z=0)$].
Let's assume that $A(z,T)$ remain independent of T then the solution of (1) is

$$A = \sqrt{P_0}\exp(i\gamma P_0 \cdot z) \quad \text{where } P_0 \text{ is the incident power.}$$

Is this solution stable against perturbations? Let's look at it : $A = (\sqrt{P_0} + a)e^{-i\gamma P_0 z}$.
Of course, this time, the perturbation $a = a(z,T)$. Therefore:

$$\partial_z A = \left[\partial_z a - i\gamma P_0(\sqrt{P_0}+a)\right]e^{-i\gamma P_0 z}$$

$$-\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2} = -\frac{1}{2}\beta_2\frac{\partial^2 a}{\partial T^2}e^{-i\gamma P_0 z}$$

$$\gamma|A|^2 A = \gamma(\sqrt{P_0}+a)(\sqrt{P_0}+a^*)\cdot(\sqrt{P_0}+a)e^{-i\gamma P_0 z} = \gamma\left\{\left[P_0 + \underbrace{|a|^2}_{\text{small}} + \sqrt{P_0}(a+a^*)\right](\sqrt{P_0}+a)\right\}e^{-i\gamma P_0 z}$$

$$= \gamma\left[P_0\sqrt{P_0} + P_0 \cdot a + P_0(a+a^*) + \underbrace{a\sqrt{P_0}(a+a^*)}_{O(a^2)}\right]\exp(-i\gamma P_0 z)$$

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finally we obtain:

$$i\partial_z a + \gamma P_0 \cancel{(\sqrt{P_0} + a)} = -\frac{1}{2}\beta_2 \frac{\partial^2 a}{\partial T^2} + \gamma\left[\cancel{P_0\sqrt{P_0}} + \cancel{P_0 a} + P_0(a+a^*)\right]$$

$$\Rightarrow \boxed{i\partial_z a = -\frac{1}{2}\beta_2 \frac{\partial^2 a}{\partial T^2} + \gamma P_0 (a+a^*)} \quad (1)$$

Note that the presence of a and a^* suggest a solution of the type:

$$a = a_1 e^{i\underbrace{(Kz - \Omega T)}_{\theta}} + a_2 e^{-i(Kz-\Omega T)}$$ and when inserted in (1):

$$\Rightarrow -K a_1 e^{i\theta} + K a_2 e^{-i\theta} = +\frac{1}{2}\beta_2\left(\Omega^2 a_1 e^{i\theta} + \Omega^2 a_2 e^{-i\theta}\right) + \gamma P\left(a_1 e^{i\theta} + a_2 e^{-i\theta} + a_1 e^{-i\theta} + a_2 e^{i\theta}\right)$$

$$\Leftrightarrow \begin{cases} \left(K + \frac{1}{2}\beta_2\Omega^2 + \gamma P\right) a_1 + \gamma P\, a_2 = 0 \\ \gamma P\, a_1 + \left(-K + \frac{1}{2}\beta_2 \Omega^2 + \gamma P\right) a_2 = 0 \end{cases}$$

which has a non-trivial solution if $\det(\ldots) = 0$

$$\Rightarrow \left(\frac{1}{2}\beta_2\Omega^2 + \gamma P - K\right)\left(\frac{1}{2}\beta_2\Omega^2 + \gamma P + K\right) - (\gamma P)^2 = 0$$

$$\Leftrightarrow \left(\frac{1}{2}\beta_2\Omega^2 + \gamma P\right)^2 - K^2 - (\gamma P)^2 = 0$$

$$\Rightarrow K^2 = \left(\frac{1}{2}\beta_2\Omega^2 + \gamma P\right)^2 - \gamma^2 P^2 = \frac{1}{4}\beta_2^2\Omega^4 + \beta_2\Omega^2\gamma P = \frac{1}{4}\beta_2^2\Omega^2\left(\Omega^2 + \overset{\operatorname{sign}(\beta_2)}{\frac{4\gamma P}{|\beta_2|}}\right)$$

$$K = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 + 4\operatorname{sign}(\beta_2)\frac{\gamma P}{|\beta_2|}} = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 + \operatorname{sign}(\beta_2)\,\underset{\hookrightarrow \frac{4\gamma P}{|\beta_2|}}{\Omega_c^2}}$$

Remember that we haven't got the information of the carrier in the envelope eq. In practice we should then have an additional factor $\exp[i(\omega_0 t - k_0 z)]$. Therefore the wavenumber and frequency of the perturbation are:

$\beta_0 \pm K$ and $\omega_0 \pm \Omega$

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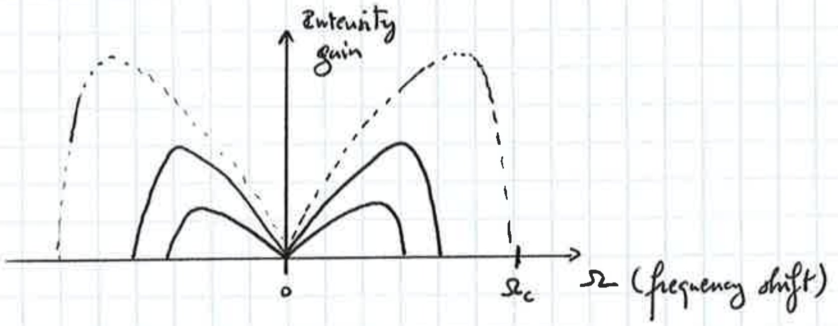
It is then clear that if $\sqrt{-\Omega^2 + \mathrm{sign}(\beta_2)\,\Omega_c^2}$ becomes imaginary, then there will be an exponential growth of the perturbation !

<u>We can draw the gain</u>. * we need the arg. of $\sqrt{\cdots}$ to be < 0, therefore $\beta_2 < 0$ and $\Omega < \Omega_c$

* we need a factor 2 to convert g into power gain.

$$\Rightarrow \quad g(\Omega) = |\beta_2 \Omega| \sqrt{\Omega_c^2 - \Omega^2}$$

$$\boxed{g(\Omega) = |\beta_2|\,\Omega \sqrt{\frac{4\gamma P_0}{|\beta_2|} - \Omega^2}}$$



Note that this curve goes through a max. which is given by

$$\frac{dg}{d\Omega} = 0 = |\beta_2| \sqrt{\frac{4\gamma P_0}{|\beta_2|} - \Omega^2} - |\beta_2|\,\Omega \frac{(-\Omega)}{\sqrt{\frac{4\gamma P_0}{|\beta_2|} - \Omega_c^2}}$$

$$\Leftrightarrow \quad \boxed{\Omega_{max} = \pm \sqrt{\frac{2\gamma P_0}{|\beta_2|}}}$$

and the peak at max: $\boxed{g(\Omega_{max}) = 2\gamma P_0}$

Even a simple CW pump can evolve into a periodic pulse train as it propagates along the fiber ! Note that the linear stability analysis that we just did only describes the initial development of the instability. This cannot grow indefinitely otherwise the linear stability analysis breaks down ! In fact there can be a cascaded process leading to multiple sidebands located at $\omega_0 \pm m\Omega$, with $m \in \mathbb{N}$

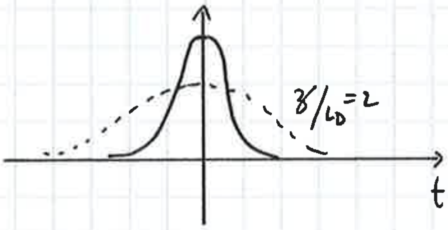
Fiber solitons

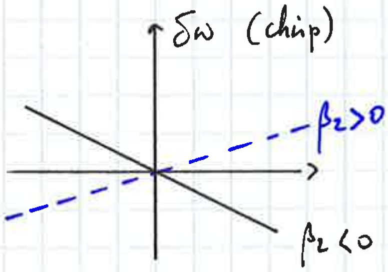
As we just saw the NLSE $i\partial_z A = -\frac{1}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A$ can have a very peculiar behavior when $\beta_2 < 0$ (anomalous regime).

Actually for $\beta_2 < 0$ there is ▲ particular pulselike solutions that either do not change during the propagation or follow a periodic pattern : optical soliton.

discovery of soliton
• Scott Russell → observation of a wave that was traveling undistorted for a very long distance (km) in a canal close to Edinburgh. (1834)
• Mollenauer : optical soliton (1988)

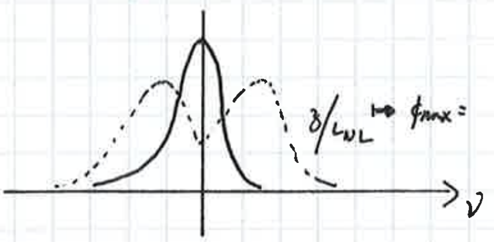
Rappel ▲ only time taken into account :

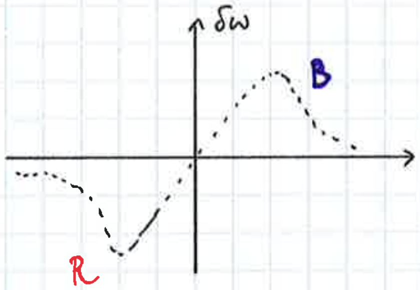




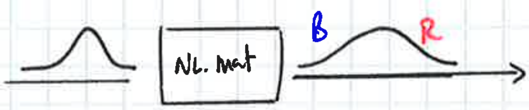
(Red faster than blue)
(Walking in the spectral domain)

▲ only non linearity taken into account.





clearly for $\beta_2 < 0$ there could be a balance between disp. & NL !



Actually we see that for a Gaussian pulse the balance cannot be perfect. In fact there exists another type of pulse described by an hyperbolic secant:

$$A(0,t) = \frac{A_0}{\cosh(t/\tau)} = A_0 \operatorname{sech}(t/\tau)$$

$$I_0(0,t) = \frac{A_0^2}{\cosh^2(t/\tau)}$$

this is the soliton.