Propagation of pulses in fibers

NLSE $$i\partial_z A = -\frac{i\alpha}{2}A - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A$$

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}} \quad \text{and} \quad A_{eff} = \frac{\left(\iint |F(x,y)|^2 \, dx\, dy\right)^2}{\iint |F(x,y)|^4 \, dx\, dy}$$

$$T = t - z/v_G$$

with v_G the group velocity

(1) Effects of dispersion:

$$i\partial_z A = -\frac{1}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} \qquad (1)$$

Note that we can eliminate the effects of loss by using the natural damping $\exp\left(-\frac{\alpha z}{2}\right)$ and replace $A(z,T)$ by $U(z,T)$ such that

$$A(z,T) = \sqrt{P_0}\, \exp\left(-\frac{\alpha z}{2}\right) U(z,T)$$

This can readily (and exactly!) be solved in the frequency domain by using the Fourier transform

$$A(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{A}(z,\omega)\, \exp(i\omega T)\, d\omega$$

the phase of each spectral component is affected differently.

then (1) becomes:

$$i\partial_z \tilde{A} = \frac{1}{2}\beta_2 \omega^2 \tilde{A} \Rightarrow \tilde{A}(z,\omega) = \tilde{A}(0,\omega)\, e^{\left(-\frac{i}{2}\beta_2 \omega^2 z\right)}$$

where $\tilde{A}(0,\omega)$ is the spectrum of the input pulse $\tilde{A}(0,\omega) = \int_{-\infty}^{+\infty} A(0,T)\, e^{-i\omega T}\, dT$

The solution of (1) is then:

$$\boxed{A(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{A}(0,\omega)\, \exp\left(i\omega T - \frac{i}{2}\beta_2 \omega^2 z\right) d\omega}$$

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Effects on a Gaussian pulse

Let us consider that the initial pulse is Gaussian : $A(0,T) = \exp\left(-\frac{T^2}{2T_0^2}\right)$

(T_0 : half-width at 1/e-intensity point)

after propagation the pulse becomes (see exercises) :

$$A(z,T) = \frac{T_0}{\sqrt{T_0^2 + i\beta_2 z}} \exp\left[-\frac{T^2}{2(T_0^2 + i\beta_2 z)}\right]$$ which is still a Gaussian pulse.

Note that

$$\exp\left[-\frac{T^2}{2(T_0^2+i\beta_2 z)}\right] = \underbrace{\exp\left[-\frac{T_0^2 T^2}{2(T_0^4+\beta_2^2 z^2)}\right]}_{\text{the pulse is getting broader! Its duration is now given by}} \underbrace{\exp\left[+i\frac{k_2 z T^2}{2(T_0^4+\beta_2^2 z^2)}\right]}_{\text{appearance of a phase term: the initially unchirped pulse becomes chirped!}}$$

$$T^2(z) = \frac{T_0^4 + \beta_2^2 z^2}{T_0^2} \longrightarrow \boxed{T(z) = T_0\sqrt{1+\left(\frac{z}{L_D}\right)^2}}$$

dispersive length $\boxed{L_D = \frac{T_0^2}{|\beta_2|}}$

Note that L_D was somehow already present in the eq. (1)

$$\underbrace{i\partial_z A} = -\frac{1}{2}\beta_2 \frac{\partial^2 A}{\partial T^2}$$

$$\frac{[A]}{\text{Length}} = [\beta_2]\frac{[A]}{(\text{Time})^2} \Rightarrow \text{typical length } L_D = \frac{T_0^2}{|\beta_2|}$$

Chirp : clearly the initially unchirped pulse (no modulation of the phase) acquires a chirp as it propagate :

$$A(z,T) = A(0,T)\exp\left[i\phi(z,T)\right] \quad \text{and} \quad \phi(z,T) = \frac{+\,\text{sign}(\beta_2)\,(z/L_D)}{1+(z/L_D)^2}\frac{T^2}{2T_0^2} + \frac{1}{2}\tan^{-1}\left(\frac{z}{L_D}\right)$$

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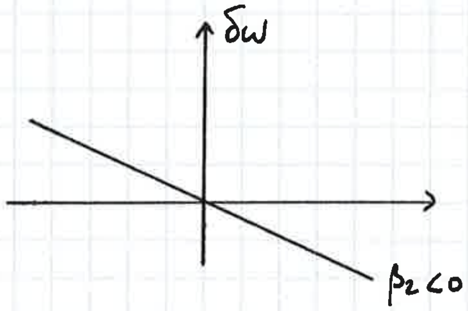
MAOT master programme in advanced optical technologies the time dependence of $\phi(z,T)$ implies that the instantaneous frequency differs across the pulse from the central frequency ω_0. this difference $\delta\omega$ is $\left(\frac{\partial\phi}{\partial T}\right)$ from the definition of the field : $E(t) = E_0 \exp(i\omega_0 t) + c.c.$

$$\Rightarrow \boxed{\delta\omega = \frac{\partial\phi}{\partial T} = \frac{\operatorname{sign}(\beta_2)\,(z/L_D)}{1+(z/L_D)^2}\,\frac{T}{T_0^2}}$$

the frequency changes linearly across the pulse $\rightarrow$ $\boxed{\text{linear chirp.}}$

for $\beta_2 > 0$ (normal dispersion regime) $\delta\omega < 0$ for leading edge of the pulse ($T<0$) and it increases linearly.

$\beta_2 < 0$ (anomalous dispersion regime) $\delta\omega > 0$ for leading edge and decreases.



Non linear phase-shift (Self-phase modulation)

Let us now assume that we do not have any dispersive effects.

$$i\partial_z A = [illegible] + \gamma |A|^2 A \quad (1)$$

~~and if we introduce U(z,T) such that A(z,T) = $\sqrt{P_0}\, e^{-\frac{\alpha z}{2}}\, U(z,T)$ the eq becomes~~

[illegible]

$$A = \sqrt{P_0}\, U \Rightarrow i\partial_z U = \gamma P_0 |U|^2 U \quad (2) \Rightarrow \text{Non linear length } \boxed{L_{NL} = \frac{1}{\gamma P_0}}$$

peak power

If we know introduce $U = V e^{i\phi_{NL}}$ and insert in (2) we obtain:

$$i \frac{\partial V}{\partial z} e^{i\phi_{NL}} - \frac{\partial \phi_{NL}}{\partial z} \cdot V e^{i\phi_{NL}} = \frac{V^2}{L_{NL}} \cdot V e^{i\phi_{NL}}$$

$\Rightarrow$ Real part:

$$-\frac{\partial \phi_{NL}}{\partial z} = \frac{V^2}{L_{NL}}$$

Imaginary part: $\frac{\partial V}{\partial z} = 0 \leftarrow$ the amplitude doesn't change during the propagation!

Therefore the eq. for the phase is readily solved:

$$\phi_{NL}(z) = -\frac{V^2}{L_{NL}} \cdot z =$$

Finally the solution of (2) is $\boxed{U(z,T) = U(0,T) \exp\left[+i\phi_{NL}(z,T)\right]}$

and $\phi_{NL} = -|U(0,T)|^2 \frac{z}{L_{NL}}$ intensity-dependant phase-shift but the pulse shape (in time) doesnot change! (SPM)

As previously (dispersive case) we can look at the effect of SPM on the chirp :

$$\delta\omega = \frac{\partial\phi}{\partial T} = -\left(\frac{\gamma}{L_{NL}}\right)\frac{\partial}{\partial T}|U(0,T)|^2$$

of course $\delta\omega$ depends on the initial chirp of $(U(0,T))$.

for no initial chirp.

Note that the frequency chirping $(\delta\omega)$ induced by SPM increases with the propagation distance.

$\Rightarrow$ New - frequency component are continuously generated as the pulse propagates down the fiber !

$\hookrightarrow$ the initial spectrum broadens as it propagates !

Ex: Gaussian pulse. $U(0,T) = \exp\left(-\frac{T^2}{2T_0}\right) \Rightarrow \delta\omega = \left(\frac{T}{T_0}\right)\cdot\frac{\gamma}{L_{NL}}\exp\left(-\frac{T^2}{2T_0^2}\right)$

Note that the pulse is almost linear around the center of the pulse (T=0) and positive. We can see that for anomalous dispersion regime, the SPM induced (chirp may be balanced by the dispersive induced chirp.
In normal dispersion → not possible (same variation)

(solitonic effect)