Modern Optics: Advanced optics
FRESNEL AND FRAUNHOFER DIFFRACTION

Exercises' sheet No 3 Nov. 2017

Exercise 1 *Fresnel and Fraunhofer diffraction*

An aperture Σ in an opaque screen is illuminated by a spherical wave converging towards a point P located on a parallel plane located at a distance z behind the aperture (Fig.)

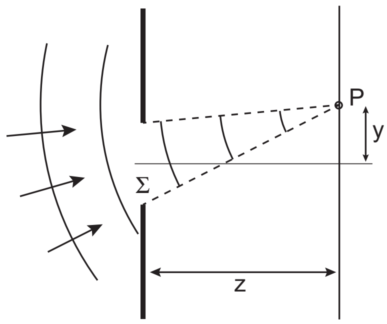

FIGURE 1 – Spherical wave converging on a screen through an aperture Σ

1. Find a quadratic-phase approximation to the illumination wavefront in the plane of the aperture, assuming that the coordinates of P in the (x, y) plane are $(0, Y)$.
2. Assuming *Fresnel* diffraction from the plane of aperture to the plane containing P, show that in the above case the observed intensity distribution is the *Fraunhofer* diffraction pattern of the aperture, centered on the point P.

Exercise 2 *Fraunhofer integral - An alternative derivation*

Spectral method is a powerful method that is commonly used to integrate ordinary differential equations (ODE) as well as partial differential equations (PDE). It is based on the tight relationship between real space and frequency domain with the help of the Fourier transform. We propose here to use such a method to integrate the complex Helmholtz equation for the propagation of a wave in the paraxial approximation

$$\triangle_\perp \psi + 2ik\partial_z \psi = 0 \tag{1}$$

where $\triangle_\perp$ is the transverse Laplacian operator.

1. Use the Fourier transfrom to write the complex Helmholtz equation in the frequency domain.
2. Fourier transform back the result of the differential equation obtained on question 1.
3. Compare the result with the Fraunhofer integral presented in the lecture.

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