Modern Optics: Advanced optics
SCALAR OPTICAL WAVES

Exercises' sheet No 2 Oct. 2017

Exercise 1 *Fresnel Coefficients*

In the lecture we consider the propagation of an EM wave between two medium determined by their respective refracive index n_1 and n_2 (Fig. 1). In this case we have $n_1 > n_2$.

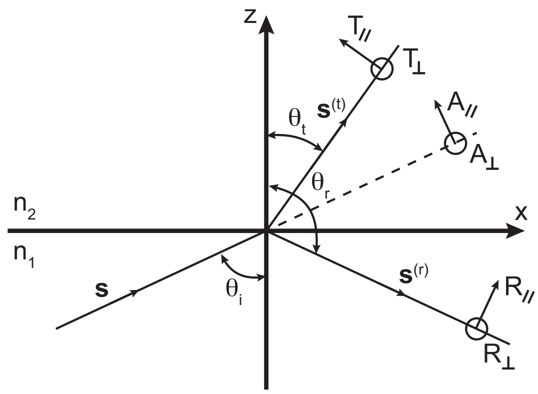

FIGURE 1 – Transmission and reflection of a wave incident to a transparent surface seprating two material characterised by their refractive index n_1 and n_2.

Let assume that the electric field of the incoming wave is given by

$$E_x^{(i)} = -A_{\parallel} \cos\theta_i e^{-i\tau_i} \tag{1a}$$
$$E_y^{(i)} = A_{\perp} e^{-i\tau_i} \tag{1b}$$
$$E_z^{(i)} = A_{\parallel} \sin\theta_i e^{-i\tau_i} \tag{1c}$$

where A is the amplitude, θ_i is the angle of incidence and

$$\tau_i = \omega\left(t - \frac{\mathbf{r}\cdot\mathbf{s}^{(i)}}{v_1}\right) = \omega\left(t - \frac{x\sin\theta_i + z\cos\theta_i}{v_1}\right)$$

1. Write the component of the magnetic field $\mathbf{H}^{(i)}$ of the incident wave.
2. What are the fields' components for the transmitted and reflected beams ?
3. Considering that the tangential components of the fields should be continuous at a boundary derive the relations

$$(A_{\parallel} - R_{\parallel})\cos\theta_i = T_{\parallel}\cos\theta_i \tag{2a}$$
$$A_{\perp} + R_{\perp} = T_{\perp} \tag{2b}$$
$$\sqrt{\epsilon_1}\,(A_{\perp} - R_{\perp})\cos\theta_i = \sqrt{\epsilon_2}\,T_{\perp}\cos\theta_t \tag{2c}$$
$$(A_{\parallel} + R_{\parallel})\sqrt{\epsilon_1} = T_{\parallel}\sqrt{\epsilon_2} \tag{2d}$$

   where $n = \sqrt{\epsilon}$.

4. Express the Fresnel relations by using the set of eq. (2).

Exercise 2 *Gaussian beam*

In the paraxial approximation the Helmholtz equation is given by

$$\triangle_\perp F + 2ik\partial_Z F = 0 \tag{1}$$

where $\triangle_\perp$ is the transverse Laplacian operator.

1. Write the conditions on $p(z)$ and $q(z)$ such that the Gaussian beam defined by

$$\psi(r) = e^{-i\left[p(z) + \frac{kr^2}{2q(z)}\right]} \tag{2}$$

2. Solve the condition for $q(z)$ and deduce that the Gaussian beam radially decays as $\exp\left(-\frac{kr^2}{2Z_R}\right)$ where Z_R is called the Rayleigh length.
3. Considering that the field has been reduced to $1/e$ at the waist $r = w_0$, derive the expression for the Rayleigh length

$$Z_R = \frac{\pi w_0^2}{\lambda} \tag{3}$$

4. Deduce from $q(z)$ the evolution of the radius of the beam $w(z)$ and the radius of curvature of its wavefront $R(z)$ considering that q can be expressed as

$$\frac{1}{q} = \frac{1}{R(z)} - \frac{\lambda}{\pi w(z)^2} \tag{4}$$