Laser & Applications
Short pulses

Exercises' sheet No 2 April 2017

Exercise 1 *Kerr-lens effect*

the goal of this exercise is to determine order of magnitude for the Kerr-lens that is self-created when a pulse propagates through a non-centro symmetric non-linear material. Such material exhibit an intensity dependent refractive index

$$n(I) = n_0 + n_2 I \tag{1}$$

where n_0 and n_2 are respectivally the refractive index and the nonlinear refractive index of the material. I is the intensity of the beam. Here we consider a piece of glass (BK7). Its refractive index at 800 nm is $n_0 = 1.51$ and its nonlinear refractive index is $n_2 = 5 \times 10^{-15}$ cm^2/W.

1. We considere a 100 fs laser pulse at 550 nm. The beam is spatially fundamental (TEM_{00}) with a beam waist of 5 mm. The phase front of beam is consideres plane. Show that when the pulse arrives on the BK7 plate (thickness is $L = 5$ mm) it experiences a focusing effect similar to the one of a lens, for which you will calculate the focal distance f for several pulse energy from 1 nJ to 10 μJ.
2. As the beam propagates through the plate it acquires a nonlinear phase-shift

$$\phi(t) = -\frac{2\pi n_2 I(t)}{\lambda} L \tag{2}$$

   Considering that the pulse is either Gaussian ($\exp\left(-t^2/\tau_0^2\right)$) or hyperbolic secant $\mathrm{sech}^2\left(t/\tau_0\right)$, where τ_0 is the pulse duration calculate the instantaneous frequency $\omega(t)$ at the output of the plate.

$$\omega(t) = \frac{d\phi(t)}{dt} \tag{3}$$

   When can observe an up- or a down-chirp? *Note : Assume that the beam remains collimated over the whole length of the plate and do not consider the effect of group velocity dispersion.*
3. What is the spectrum of the Gaussian pulse at the entrance and after its propagation through the plate? You can view the effect of the chirp as a small correction.
4. Same question for the temporal shape of the pulse.

Exercise 2 *Influence of dispersion on pulses*

In this exercise we focus on the linear propagation of a pulse inside a dispersive material. The pulse shape can be mathematically described as

$$E(z,t) = A(z,t)\ e^{i(\omega_0 t - kz)} \tag{1}$$

where ω_0 is the angular frequency of the pulse and $k(\omega) = n(\omega)\omega/c$ is its wave-number. The equation of propagation of the pulse can be writen as

$$i\frac{\partial A}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} \tag{2}$$

where the coefficient β_2 is given by

$$\beta_2 = \frac{d^2k}{d\omega^2} \tag{3}$$

1. Show that if the equation (2) has an exact solution if it is integrated in the spectral domain.
2. Show that in the case of a Gaussian beam

$$A(z=0,T) = exp\left(-\frac{T^2}{2T_0^2}\right) \tag{4}$$

   the solution after a propogation over z is

$$A(z,T) = \frac{T_0}{\sqrt{T_0^2 - i\beta_2 z}} \exp\left[-\frac{T^2}{2\left(T_0^2 - i\beta_2 z\right)}\right] \tag{5}$$

3. From eq. (5), we find the the pulse actually propagates without and maintain its shape but its pulse width changes according to

$$T(z) = T_0\sqrt{1+\left(\frac{z}{L_D}\right)^2} \tag{6}$$

4. Describe this formula (Eq. (6)) as an analogy to the evolution of spatial Gaussian beam. Which parameters correspond to beam width (w_0) at the focus point, the radius of curvature of the phase-front, the Rayleigh length and the divergence angle repsectively.
5. What is the pulse width of a pulse traveling thought a plate (BK7) ? The initial pulse length is 100 fs and its central wavelength is 546 nm. The plate is either 5, 10 and 50 mm long. Calculate the instantaneous frequency $\omega(t) = {}^{d\phi}/_{dt}$ where $\phi(t) = Arg\left(A(z,T)\right)$. Compare the results with the ones from exercise one. When can the effect of disperion and those of the Kerr-effect balance ?