# Laser & Applications
## Z-shaped cavity

*Exercises' sheet No 1* April 2017

## <u>Exercise</u> 1 *Brewster-angled crystal*

1. Calculate the transfer matrix for a crystal with an angle of incidence $\theta_1$ (Fig. 1)

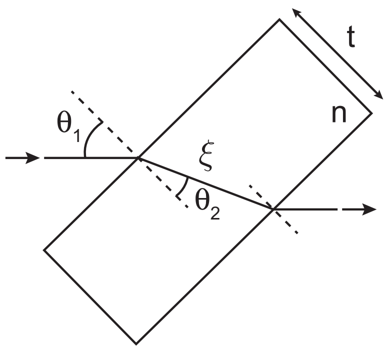


Figure 1 – Brewster-cut cyrstal

2. Express the optical path $\xi$ as a function of the physical thickness $t$ and the refractive index of the crystal $n$ when the crystal is cut at Brewster angle.
3. Show that in that case the matrices reduce to

$$M_{\text{sag.}} \begin{bmatrix} 1 & \frac{\xi}{n} \\ 0 & 1 \end{bmatrix} \tag{1a}$$

$$M_{\text{sag.}} \begin{bmatrix} 1 & \frac{\xi}{n^3} \\ 0 & 1 \end{bmatrix} \tag{1b}$$

## <u>Exercise</u> 2 *Confocal and concentric cavities*

In the lecture, we expressed the equivalent lenght for a Z-shaped cavity as

$$\mathscr{L}_{\text{sag}} = (\ell - t) + \frac{\xi}{n} - \mathcal{R}_{\text{sag.}} + 2\rho_{\text{sag.}} \tag{1a}$$

$$\mathscr{L}_{\text{tgt}} = (\ell - t) + \frac{\xi}{n^3} - \mathcal{R}_{\text{tgt.}} + 2\rho_{\text{tgt.}} \tag{1b}$$

where

$$\rho_{\text{sag.}} = \frac{f^2/\cos^2\theta}{f/\cos\theta - L} \qquad \mathcal{R}_{\text{sag.}} = R \times \cos\theta \tag{2a}$$

$$\rho_{rmtgt.} = 2\frac{f^2\cos^2\theta}{f\cos\theta - L} \qquad \mathcal{R}_{\text{sag.}} = \frac{R}{\cos\theta} \tag{2b}$$

and $R$ is the radius of curvature of the two spherical mirrors in the Z-shaped cavity (Fig. 2).

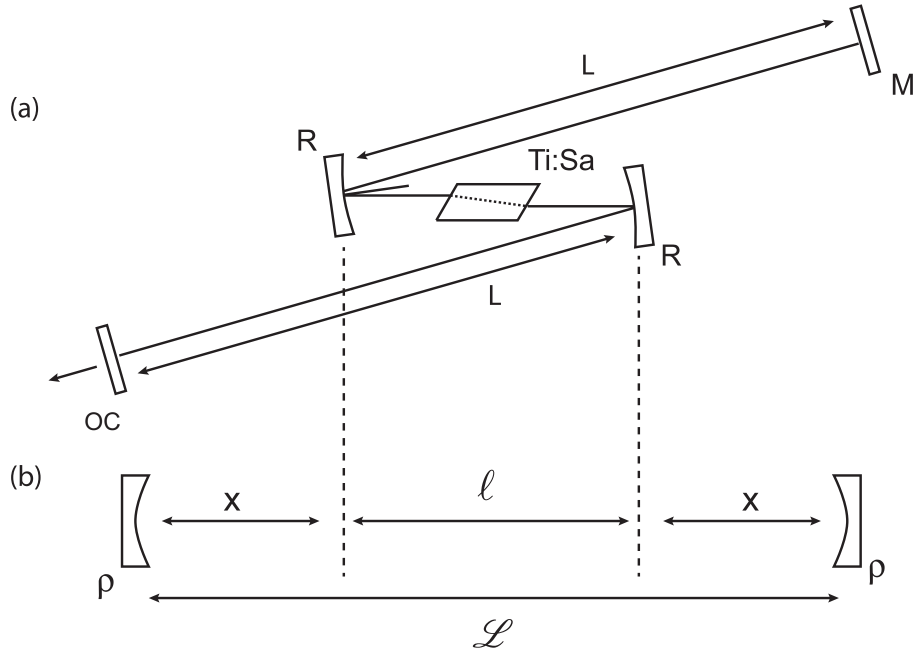


FIGURE 2 – Z-cavity and its equivalent as a spherical-spherical cavity

1. show that the angle $\theta$ such that both sagital and tangential equivalent cavities are simultaneously in a confocal configuration is given by (in the usual case where $L \gg R$)

$$\theta_{\text{conf.}} \simeq \sqrt{\frac{t\sqrt{n^2+1}}{nR}\left(\frac{1}{n}-\frac{1}{n^3}\right)}\left[1-\frac{1}{2}\frac{f^2(2L-f)}{R(f-L)^2}\right] \tag{3}$$

2. show that to have both cavities in a concentric configuration the angle is given by

$$\theta_{conc.} \simeq \sqrt{\frac{t\sqrt{n^2+1}}{nR}\left(\frac{1}{n}-\frac{1}{n^3}\right)} \tag{4}$$

3. compare both angles for the case where $L = 1$m, and the mirrors have a radius of curvature of 10 cm. The thickness of the crystal is $t = 4$ mm and its refractive index is 1.76.